DOCUMENT RESUME

ED 355 123 SE 053 494

AUTHOR	Welchman-Tischler, Rosamond
TITLE	How To Use Children's Literature To Teach Mathematics.
INSTITUTION	National Council of Teachers of Mathematics, Inc., Reston, Va.
REPORT NO	ISBN-0-87353-349-6
PUB DATE	92
NOTE	85p.
AVAILABLE FROM	National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091-1593 ($8.50).
PUB TYPE	Guides - Classroom Use - Teaching Guides (For Teacher) (052)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*Childrens Literature; Creative Activities; Elementary Education; Integrated Activities; *Interdisciplinary Approach; Manipulative Materials; Mathematical Concepts; Mathematics Education; *Mathematics Instruction; *Mathematics Skills; *Problem Solving; Review (Reexamination); Teaching Methods
IDENTIFIERS	Connections (Mathematics); NCTM Curriculum and Education Standards; *Reading Mathematics Relationship

ABSTRACT

The National Council of Teachers of Mathematics' "Curriculum and Evaluation Standards" envision an interdisciplinary approach to mathematics that allows children to communicate their ideas in the process of solving real problems. This book promotes the connection between mathematics and children's literature as part of realizing that vision. An introduction provides the rationale for this connection and describes the organization of the eight chapters of the book, the rationale for the books selected, supplementary resources, and general ideas on mathematics and children's literature. The first seven chapters present activities organized according to the following themes: (1) providing a context; (2) introducing manipulatives; (3) modeling a creative experience; (4) posing an interesting problem; (5) preparing for a concept or skill; (6) developing a concept or skill; and (7) providing a context for review. Each chapter begins with a discussion of the theme and presents activities involving stories that exemplify the themes. Each activity provides the author and publication year of the story, a summary of the story, a classroom sketch of the activity, appropriate grade levels, and possible follow-up activities. Worksheets related to particular activities are included at the end of each chapter. The eighth chapter examines ways in which mathematical thinking can be applied in the classroom experience of children's literature in general. A list of 20 books for teachers and a list of 81 books for children are provided. (MDH)

ED355123

How to USE CHILDREN'S LITERATURE TO TEACH MATHEMATICS

Rosamond Welchman-Tischler

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☒ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL IN **OTHER THAN PAPER COPY** HAS BEEN GRANTED BY

Jean Carpenter

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

BEST COPY AVAILABLE

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

HOW TO USE CHILDREN'S LITERATURE TO TEACH MATHEMATICS

Rosamond Welchman-Tischler

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Copyright © 1992 by
THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, INC.
1906 Association Drive, Reston, Virginia 22091-1593
All rights reserved

Library of Congress Cataloging-in-Publication Data:

Tischler, Rosamond Welchman
How to use children's literature to teach mathematics / Rosamond Welchman-Tischler.
p. cm.
Includes bibliographical references.
ISBN 0-87353-349-6
1. Mathematics—Study and teaching (Elementary) 2. Children's literature in mathematics education. I. Title.
QA135.5.T57 1992
372.7'044—dc20 92-30867
CIP

The publications of the National Council of Teachers of Mathematics present a variety of viewpoints. The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of the Council.

Printed in the United States of America

CONTENTS

Acknowledgments iv

Introduction 1

Summary Chart 4

Provide a Context 5

Introduce Manipulatives 13

Model a Creative Experience 37

Pose an Interesting Problem 43

Prepare for a Concept or Skill 50

Develop a Concept or Skill 57

Provide a Context for Review 63

General Approaches 66

"Integrating Mathematics into an Author Study" by Elizabeth Phillips 66

Books for Teachers 73

Books for Children 74

ACKNOWLEDGMENTS

Work on this booklet was partially supported by a grant from the National Science Foundation, Teacher Enhancement Program. I would especially like to thank Dorothy Geddes, with whom I codirected this program, and all the teachers with whom I have worked for their contributions to this booklet. The editorial board and the reviewers for the National Council of Teachers of Mathematics were also particularly helpful. Finally, I would like to thank my children, Katy and Danny, with whom I initially read most of the books discussed.

INTRODUCTION

Mathematics should be a living, exciting activity for children. It can give children a tool for solving real problems and a way of looking at, and communicating about, their world that adds understanding and insight. This is the vision described by the National Council of Teachers of Mathematics in its *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989). This booklet explores ways teachers can help children to experience the type of mathematics envisioned in the *Standards* while building on what grips the imagination of students and teachers—children's literature.

Mathematics activities should be appropriate to a child's level of development and should engage children in doing mathematics. This booklet suggests ways to provide thought-provoking mathematics experiences related to literature in a manipulative context. Sample activities lay the groundwork for the sound development of concepts and promote experimentation with ideas. Many suggestions are included for modifying activities to meet individual needs. The four generic standards proposed by NCTM are mathematics as problem solving, mathematics as communication, mathematics as reasoning, and mathematical connections. All these underlie the approach to mathematics teaching in this booklet. Promoting the connection between mathematics and children's literature is the fundamental purpose of the booklet, problem solving and reasoning play an important role in most activities, and communication is fostered in many forms—verbal, visual, manipulative, and symbolic.

Interdisciplinary approaches to teaching, such as using the mathematics-literature connection, have many benefits. Children must find mathematical experiences interesting if they are to achieve their mathematical potential, and using literature as a springboard is one way to capture their interest. Every teacher is frustrated by the lack of time to cover all the curriculum areas to which children should be exposed. For example, teachers are expected to teach students the mathematics curriculum, literature, and higher-order thinking skills. If these topics are taught in isolation, as pictured in figure 1, more time will be required than if teachers develop the interconnections, as shown in figure 2.

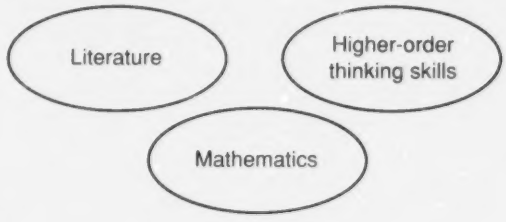


Fig. 1

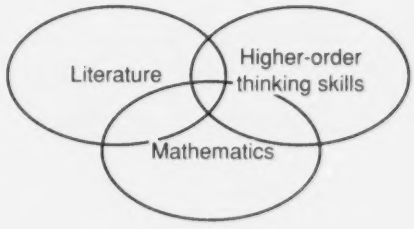


Fig. 2

Interdisciplinary studies not only save time but also add to children's insight into all curriculum areas involved—the whole is greater than the sum of the parts.

Many children's books relate to mathematics. Some are explicitly about mathematics, for example, counting books, books about shapes, and a variety of trade books that aim to teach specific mathematics concepts. Children can learn about mathematics directly from these books, which can be identified in an index of children's books by topic. See, for example, *The Wonderful World of Mathematics: A Critically Annotated List of Children's Books in Mathematics*, by Diane Thiessen and Margaret Matthias (1992). Some more general listings limit the coverage of mathematics to number. For example, in *A to Zoo, Subject Access to Children's Picture Books* (3d ed.), by Carolyn W. Lima and John A. Lima (1989), the

only entry under the heading Mathematics is "*See* Counting." (However, this resource has many listings related to mathematics under the heading Concepts, which includes the following as subheadings: distance, measurement, opposites, perspective, shape, size, speed, weight.)

Other children's books involve mathematics in more subtle ways. They might not be called "math books" by teachers or students, yet they suggest rich possibilities for extended mathematical investigations. Most of the books described in this booklet are of this second type. Although many of these titles refer to mathematical concepts, almost all stand on their own as good books, and teachers would not think of reading them only in a mathematics period. This booklet is intended to help teachers look at such books in a new light and see how they create connections between the vision of mathematics learning in the *Standards* and children's literature. It is not intended to be an exhaustive bibliography of appropriate books; rather it is intended to illustrate approaches that can be applied to many excellent children's books of the past, present, and future.

Organization

The books cited are merely examples whose uses can be adapted to the children's or the teacher's favorite books. To promote such adaptations, each of the first seven chapters of this booklet describes a type of use of children's literature to teach mathematics. The uses are classified as follows:

1. To provide a context or model for an activity with mathematical content
2. To introduce manipulatives that will be used in varied ways (not necessarily as in the story)
3. To inspire a creative mathematics experience for children
4. To pose an interesting problem
5. To prepare for a mathematics concept or skill
6. To develop or explain a mathematics concept or skill
7. To review a mathematics concept or skill

These uses are described more fully at the beginning of each chapter. Each chapter then contains examples of two to four books, with a summary of the story of each book, a sketch of proposed classroom activities and related print materials (models for manipulatives, activity cards, worksheets), and some suggested follow-up activities. The listing of Other Books contains thumbnail sketches of additional books that can be used in similar ways. Sometimes books can be used in more than one way and thus are mentioned in more than one chapter. Grade levels are suggested but are only approximate, since it is the teacher's level of questioning about the mathematics in the book, not the mathematics topic or the book itself, that determines the appropriateness of the activity for a given group of children. Also, for upper-grade students with limited reading skills, teachers might find that a book written for much younger children can be refreshing and build confidence.

The chart on page 4 summarizes the types of uses, the books selected, the approximate grade levels, and the mathematics topics discussed. Note that many books relate to a variety of mathematics topics. Which topics a teacher should emphasize will depend on the grade level and the curriculum topics under consideration.

Specific techniques that are described in one chapter can often be applied to other books, and in fact, to the teaching of mathematics in other contexts. For example, the poke-through cards suggested for *Frog and Toad Are Friends* in chapter 1 can be applied to any question with multiple-choice answers. The little fold-up books suggested for *Caps for Sale* can be designed by teachers or children for a wide variety of topics.

Chapter 8 has a different format from the other seven chapters. The first part describes some general strategies for looking at children's literature from a mathematical point of view; then Elizabeth Phillips, a first-grade teacher at P.S. 321 in Brooklyn, New York, has written about the ways she has made connections between mathematics and the study of an author and his books.

Rationale for Selection

Many books could have been chosen as examples. All featured in this booklet are readily available. The books were selected to represent a wide range of grade levels and a wide range of mathematics topics. All are books that children enjoy reading.

Many children's books have significant, but brief, allusions to mathematical ideas. Of course these should be recognized and exploited, but dwelling on a mathematical "aside" for too long might interrupt the flow of the book. In general, the examples chosen for

this booklet are suitable not only for passing comment but for expansion into a subsequent mathematics lesson or activity.

Further Resources

Recent elementary school textbook authors have made significant efforts to use children's literature in their mathematics programs. Some series have stories written exclusively for the mathematics program. (The storybooks by Frédérique and Papy—*Graph Games* (1971) is an example—written for the Comprehensive School Mathematics Program (CSMP) for elementary school and published by CEMREL, and the Thinking Story Books in the Real Math program published by Open Court (1985) were leaders in this trend.) These stories are a valuable resource for teachers and should be used in conjunction with the more general types of children's literature described in this book.

Other connections between mathematics and children's literature are described in *Beyond Numbers: The Mathematics Literature Connection*, by John P. Madison and Roslynn Seidenstein (1987).

General Ideas on Mathematics and Children's Literature

Some books involve attitudes toward mathematics rather than any specific mathematical topic. One is reminded of the Mock Turtle's description of "the different branches of Arithmetic—Ambition, Distraction, Uglification, and Derision" in Lewis Carroll's *Alice's Adventures in Wonderland* (1985). Some books refer explicitly to children's experiences with mathematics in school. The picture is seldom flattering. In Lewis Carroll's *Through the Looking Glass* (1972), the red and white queens give Alice a "proper examination" on arithmetic, in which Carroll captures a hilarious lack of reason in their discussion of such a presumably reasonable subject. Another example is in *Pippi Longstocking*, by Astrid Lindgren (1978). The heroine of this book (one of a series) is an irrepressible girl with extraordinary powers who lives alone with her animals in a very unconventional way. Neighboring children talk her into going to school (because she realizes that it is unfair—she cannot have a vacation unless she goes to school!), and her reaction to what is asked of her has something to say to teachers of mathematics. First she is asked what 7 and 5 makes, and she responds, "Well, if you don't know that yourself, you needn't think that I am going to tell you!" (p. 52). She is told that the expected answer is 12, and she is then asked what 8 and 4 are. When the teacher tells her that this also is 12, she replies,

> Well now, really, my dear little woman, ..., that is carrying things too far. You just said that 7 and 5 are 12. There should be some rhyme and reason to things, even in school. Furthermore, if you are so childishly interested in that foolishness, why don't you sit down in a corner by yourself and do arithmetic and leave us alone so that we can play tag? (P. 53)

(Reprinted courtesy of Viking Penguin, a division of Penguin Books USA Inc.)

We can draw a lesson from this vignette—that children's mathematical experience should be based on real problems, of interest both to children and to teachers. Children's literature, used in some of the ways described in this booklet, can show us a way to provide this kind of mathematical experience.

Summary Chart

Use	Book	Grade Level	Mathematics Topic
1. **Provide a Context**	*Frog and Toad Are Friends*	K–2	Classification, reasoning
	Stone Soup: An Old Tale	K–4	Classification, graphing, measurement
	The Hundred Penny Box	3–5	Graphing, sampling, measurement of time
Other Books	*Pancakes for Breakfast, Chocolate Chip Cookie Contest*—food and cooking; *A Chair for My Mother*—counting, earning, and spending money; *Paper Crane, Sadako and the Thousand Paper Cranes*—origami; *Over in the Meadow, I Know an Old Lady Who Swallowed a Fly*—songs; *Caps for Sale*—mysteries; books about patchwork		
2. **Introduce Manipulatives**	*Ten Apples Up on Top*	1–2	Grouping by tens, place value
	Caps for Sale	K–2	Patterning, counting, addition, subtraction
	More Spaghetti, I Say!	K–3	Counting, patterning, graphing, addition, measurement, place value, shopping
	Millions of Cats, So Many Cats!	K–2	Classification, graphing, estimation, logical reasoning
Other Books	*Ten Bears in My Bed; Swimmy; Count on Your Fingers African Style; Little Red Caboose, Freight Train*—trains; books about squirrels; *Ruth's Bake Shop; All the Pretty Horses; One Fish Two Fish Red Fish Blue Fish*		
3. **Model a Creative Experience**	*Anno's Counting Book*	K–1	Counting, classification
	Ed Emberley's Picture Pie	3–5	Fractions, mixed numbers, symmetry
	One, Two, Where's My Shoe?	K–2	Matching shapes, shapes in the environment
Other Books	Counting books; shape books; stories to complete; books on quilts; *Twelve Days of Christmas*		
4. **Pose a Problem**	*Twelve Days of Chistmas*	2–6	Counting, number pattterns
	The Witches, Matilda	4–6	Rate, forming and verifying hypotheses, graphs
	A Grain of Rice	4–6	Number patterns, calculators, large numbers, exponents
Other Books	Books by Anno; *Math-a-magic; And Then What Happened, Paul Revere?*—history; *Jesse Owens*—sports biographies; *Alice in Wonderland, The Borrowers, Gulliver's Travels, Stuart Little*—change of scale		
5. **Prepare for a Concept or Skill**	*The Doorbell Rang*	1–2	Division
	Reflections, Round Trip	2–4	Symmetry (line and rotational)
	Charlie and the Great Glass Elevator	4–6	Negative numbers
Other Books	Concept books; *Little Red Hen, Gingerbread Man*—sequence; *Too Much Noise*; number books; *Magic Mirror Tricks*		
6. **Develop a Concept or Skill**	*So Many Cats*	K–2	Addition, subtraction
	How Big Is a Foot?	1–3	Measurement of length
Other Books	*Anno's Mysterious Multiplying Jar; Ed Emberley's Picture Pie; Clocks and More Clocks; The Doorbell Rang*		
7. **Provide a Context for Review**	*Arthur's Funny Money*	2–3	Addition, subtraction
	How Much Is a Million?	3–6	Large numbers, multiplication, estimation
Other Books	*Alexander Who Used to Be Rich Last Sunday; The 500 Hats of Bartholemew Cubbins; Where the Sidewalk Ends; A Book of Nonsense*		
8. **General Approaches**	All of Tomie dePaola's work		

1

PROVIDE A CONTEXT

Some examples of children's literature suggest an actual classroom activity related to mathematics, which can be adapted almost directly from the book. This use of a book is especially valuable for making children aware of the many ways mathematics is used in their everyday lives and thus promotes the NCTM curriculum standard on mathematical connections.

Sometimes stories with a mathematics-related plot stand by themselves, and one wouldn't want to modify them in any way; others lend themselves to repetition, with slight modifications. The story in *Frog and Toad Are Friends* (Lobel 1970), for example, suggests a classroom reenactment of a search for a lost button, and endless variations can be suggested by the children. This activity exercises many linguistic skills, both talking and listening, as envisioned in the NCTM curriculum standard on mathematics as communication.

Many children's books focus on everyday activities (e.g., cooking, shopping) that use a variety of mathematical skills and approaches. An example is a retelling of the old folk tale *Stone Soup* (Brown 1986), which is especially appropriate for introducing a classroom soup-cooking activity, because in acting out the story, children can play the parts of the villagers who contribute the ingredients for the soup. This activity gives the teacher many options for the development of mathematical ideas. The teacher can select which of the many mathematical aspects that could be highlighted to suit the needs and interests of the children. The soup-cooking activity can also be related to several curriculum areas besides mathematics and offers the benefits of interdisciplinary activities in the motivation of the students, the efficient use of classroom time (because more than one curriculum area is involved), and the appreciation of connections among curriculum areas.

In the third example in this chapter, *The Hundred Penny Box* (Mathis 1986), pennies are used to symbolize the years in an old woman's life. This book contains an emotionally compelling treatment of the problems of old age and the changing relationships among mother, daughter, and grandson. In any use of children's literature to teach mathematics, the fundamental impression of the story should not be distorted by an undue emphasis on a mathematical aspect. However, the image of pennies in the hundred-penny box is strong enough to be developed into a classroom activity after the emotional impact of the book has been felt.

Books based on mathematics-related activities—such as cooking, shopping, making things, solving mysteries by logical reasoning from clues, or music—will supply more possibilities for the type of use discussed in this chapter.

Frog and Toad Are Friends

("The Lost Button" chapter)
Arnold Lobel (1970)

Summary of the Story

Frog and Toad go for a walk. Frog loses a button, and he and his friends search for it. They find many buttons, and each time, one of Frog's friends asks, "Is this your button?" But Frog angrily says, "No, that is not my button! That button is ____, but my button was ____." This statement is repeated for various attributes of the buttons. Finally, as is so often the case in real life, Frog finds his button at home—he lost it before he left. To make up for being so angry with the friend who was trying to help him, he sews all the found buttons on a jacket and gives it to Toad.

Classroom Sketch (Grades K–2)

Classification
Reasoning

Each group of children will need a collection of buttons. Beforehand, select a few possible lost buttons, placing identical copies of these buttons in each of the children's collections.

1. Read the story to the children. Pass out the collections of buttons. Reread the story, letting children act out the parts of the helpful animals.

2. Suggest trying the story again; this time with a different lost button. You will play the part of Frog, and the children will be the helpful friends. Explain that the button you have "lost" is actually in your hand. Ask children to select and hold up a button and to say, as in the story, "Is this your button?" You reply, "No, that is not my button! That button is ____ , but my button is ____ ," filling in the attributes as appropriate. (It helps to have more than one possible lost button in your hand, in case a child guesses too quickly!) Have children keep suggesting buttons until one of them holds up the correct button.

While they are playing this game, encourage the children to listen to each other. Some children might not listen or remember all of the clues, but other children will probably volunteer, "It couldn't be that button because Frog said that it had two holes, and that one has four!"

3. When the children understand the game, let one of them play the role of Frog. They may need help at first in describing the differences between buttons.

Follow-up: Material for Individual Use

"Which One Is Different?" poke-through cards. This material is self-checking and has the added benefit of acquainting children with a multiple-choice format. Use squares of cardboard, about 10 cm on a side, and divide them into four sections as shown in figure 1.1. Punch holes in the four corners. Glue four buttons, one of which is different in some way, on the card. Make a mark the back of the card next to the hole that corresponds to the different button. Children use the cards by deciding which button is different, sticking a coffee stirrer through the corresponding hole, stating the reason for their decision, and then turning the card over to check. Sometimes there will be more than one correct answer. If this is so, just mark more than one correct answer on the back. Encourage children to find more correct answers and mark their choices on the back if they are well explained.

Note: For more ideas on using this book to teach mathematics, see "Mathematics from Children's Literature" by Rosamond Welchman Tischler (1988, pp. 42–47).

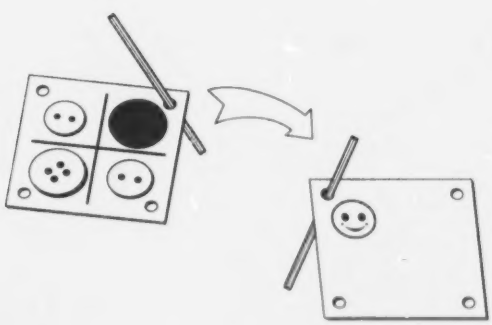


Fig. 1.1

Stone Soup: An Old Tale

Marcia Brown (1986)
First published in 1947

Summary of the Story

Three hungry soldiers arrive in a village where the residents fear strangers. When the soldiers ask the villagers for food, there is nothing to be spared. So the soldiers explain to the villagers that they will make "stone soup." The villagers provide a fire, a pot, water, and three stones to put in the pot. The soldiers say that of course the soup will be delicious, but it would be so much better with a carrot, and a woman (who is full of curiosity about how a soup made from stones will taste) brings carrots. The same pattern is repeated for cabbage, potatoes, barley, beef,.... Soon

BEST COPY AVAILABLE

the soup is full of delicious things, and the whole village is treated to a banquet, as they marvel about how such a soup could be made from just three stones.

Classroom Sketch (Grades K–4)

Classification
Graphing
Measurement

1. Read the story to the children. Suggest that the class make stone soup. You, the teacher, will provide the pot, water, and three stones, and the children can be the villagers. Ask the children to bring a vegetable from home. (Have a collection of interesting vegetables on hand for children who forget or who are unable to bring something.)
2. When the vegetables arrive, put each one on a small paper plate on the floor or a large table top. Encourage the children to discuss them. It is likely that children will mention the following (in their own words):
   - Classification by type (carrot, potato, bean,...)
   - Classification by part of the plant (leaf, root, stem, fruit,...)
   - Comparison of size (length, height, weight, volume,...) [If desired, have measuring equipment so that children can estimate which vegetable is longest around or has the greatest volume or is the heaviest and then check. Older children might be exposed to the concept of density by investigating whether the vegetable with the greatest volume is also the vegetable with the greatest weight.]
   - Classification by personal experience, likes and dislikes, and so on

Each type of classification can be expressed in different ways, in particular, by grouping the vegetable plates in a Venn diagram formed by large loops of thick yarn (fig. 1.2), or by arranging them in rows on a grid to make a bar graph (fig.1.3). (It is especially useful to have such a grid made from a plastic window shade.)

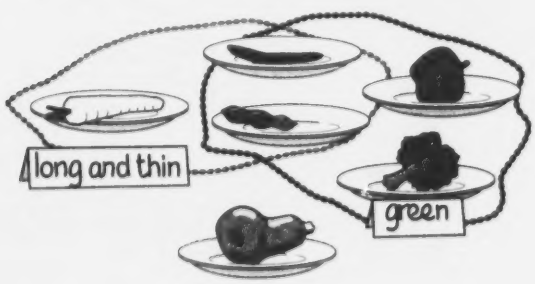


Fig. 1.2

Fig. 1.3

3. Suggest making a record of the classification by type of vegetable so that the class can use the vegetables for the soup but still know what they were. Give each child a square of paper on which to draw a picture of one vegetable that will go in the soup. When each child puts the drawing on the grid in place of the vegetable and its plate, the squares can be glued onto a large sheet of paper to make a bar graph fig. 1.4). If you want the children to take the recipe home, prepare a blank grid and have them copy the labels of the pictorial graph onto it, then shade in squares to correspond to the pasted-on pictures (fig. 1.5). In this way, children can experience three steps in the progression needed to understand bar graphs—one made with physical objects, one made with pictures of the objects, and one in which shaded squares represent the objects.
4. Make the soup! Children should cut the vegetables and put them in the pot. This process can lead to discussion of a wide range of mathematical questions. Here is a sample:

- Vegetables like potatoes are often cut in little cubes. *How can you make these cubes?*

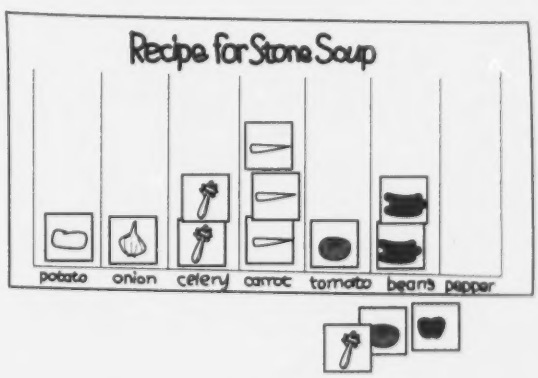


Fig. 1.4

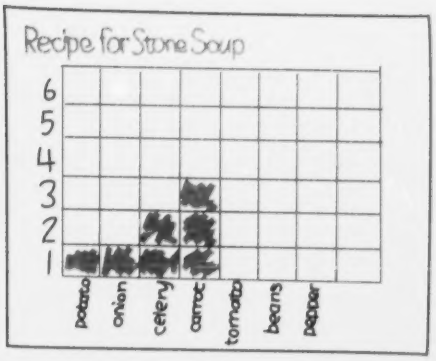


Fig. 1.5

- *What shapes can you get from cutting a carrot? Certainly you can get a circular region as the cross section, but what if you cut the carrot on a slant?* [You will get a region whose boundary is an *oval* or an *ellipse*.] *How could you get pieces whose faces have triangular outlines?*
- The vegetables will taste best if they are all cut into pieces of approximately the same size. *How can this be done?*
- *How many pieces do you get if you first cut a potato in half, then in half again, and again,...?*
- *If you want 1/16 of a carrot, how can you approximate it?*
- *How can you measure units of time?* [Consider a home-made sand clock. Make a small hole in the bottom of two clear plastic cups with a knitting needle heated over a flame. Tape the two cups together at their bottoms. Glue cardboard circles on the tops of the two cups, after filling one of them about half full of salt or sand. See the illustration in figure 1.6. A child can be responsible for testing the soup each time the sand runs out.]

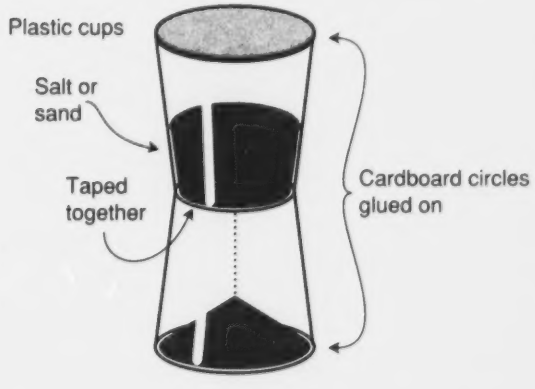


Fig. 1.6

This kind of soup always seems to taste good even if the recipe is a bit haphazard. Here are some suggestions for success: Start with a few bouillon cubes in the water or perhaps just salt, pepper, and a few other herbs. Try to put in the slower cooking vegetables first (carrots, onions, celery, potatoes), then vegetables such as beans and peas, and finally, leafy vegetables such as spinach or cabbage. (If the children are interested, they could investigate how long each type of vegetable should be cooked and how the size of the pieces affects the cooking time.)

Follow-up: Serving, Tasting, and Graphing

When the soup is done, serve it and taste it! This procedure leads to several more questions:

- *How many people can the soup serve?* Estimate how many cups are in the pot, and discuss ways to check the estimate short of serving it. [For example, fill a similar pot to the same height with water and measure.]
- Try an opinion poll. Give a list of possible descriptions (delicious, awful, so-so, bland, interesting,...) and have children make a graph showing how many think each description is appropriate.
- Do a taste test. Get some impartial observers to taste the stone soup and compare it to a canned vegetable soup. Make a graph of which one is preferred.
- Look at the contents of a spoonful. Count how many pieces of each vegetable there are. *Does this correspond to the recipe? Why or why not?*

Follow-up: Writing and Adapting a Recipe

Write the recipe, possibly after changes suggested by taste testers. Ask the children to modify the recipe to make enough for 100 villagers, for the whole school, for just their family....

Many of the mathematical concepts arising in this recipe relate to other types of cooking. The book *Stone Soup* provides a nice context for designing a recipe in a somewhat random way. The same plan could be adapted to making fruit salad, if you do not want to cook with heat.

Follow-up: Comparison Shopping

Older children might be interested in how much their stone soup costs compared to a commercial canned soup. To determine the answer, they may need to visit a supermarket or examine a supermarket hand-out and do a considerable amount of calculation. Three bags of carrots sell for $1.00. *What is the average number of carrots in a bag? How many carrots did our recipe use?*

The Hundred Penny Box

Sharon Bell Mathis (1986)
Illustrated by Leo and Diane Dillon

Summary of the Story

In this very moving story, an old woman relives her life through her "hundred penny box," into which a penny has been put for each year of her life. With a story like this, one would not want to interfere with the emotional impact by moving directly into a mathematics activity. Yet the mathematics potential in looking at penny collections and thinking of the passage of time in their dates strongly suggests introducing some related activities soon after reading the book.

Classroom Sketch (Grades 3–5)

Graphing
Sampling
Measuring

1. Ask for contributions to a class penny collection. (If desired, children can be made responsible for counting the pennies each person contributes so that the pennies can be returned.)
2. Divide the class into groups of three or four. Distribute a share of the pennies to each group. (Children can figure out how many each group should get.) The children look at their coins and predict, on the basis of what their group has, the oldest coin, the newest coin, and the most common year.
3. Give a blank piece of graph paper to each group and ask the children to make a graph of the group's coins. (If desired, use activity sheet 1.1, where the pennies will fit exactly on the squares to form a concrete bar graph.) Each group should decide how its graph should be made. Perhaps children can group years, say 1965–69. *Compare the graphs of different groups. How are they alike? How are they different?*
4. *What will happen if all the data are combined on one graph?* Discuss with the class which group's type of graph should be made and have the children construct a class graph. Questions like the following might arise: *Why are certain years more common? What is the oldest coin? Are older ones more valuable?* A child might do some research on the value of old coins.
5. Finally, the children might start their own "hundred penny" boxes, in which they try to put a penny for each year of their lives and write about an event that happened in that year. As a class project, such a box might be made for the principal or for the school building itself.

Follow-up: Homework Assignment

Ask children to try to assemble a hundred-penny box for a parent or someone else in their family and to interview a parent about important events in their lives for each year.

Other Books

To find more children's books that can provide a context for a mathematical activity, think of the contexts in which mathematics is used. For example, cooking involves measurement, shopping involves computation, arts and crafts involve spatial sense and geometry, traditional folk songs involve sequence and counting, mysteries involve the logical interpretation of clues. These examples are organized by the type of context they represent. You can probably think of many other contexts and for each context, a number of different books.

Food and Cooking

Most of the mathematical ideas that were mentioned in *Stone Soup* could arise in many other books about cooking or eating or both. These books suggest activities with the measurement of capacity and time (through cooking), coins and the computation of value (through shopping for ingredients), and sampling and graphing (through taste tests and opinion polls). Here are two samples:

Pancakes for Breakfast (Grades Pre-K–2) This almost wordless picture book by Tomie dePaola

(1978) is about the complicated steps a woman must go through to collect the ingredients for pancakes. The recipe is included; so reading the book could be followed naturally by a cooking activity.

The Chocolate Chip Cookie Contest. A boy enters and wins a chocolate chip cookie contest in this book by Barbara Douglas (1985). The book suggests collecting data about a taste test of chocolate chip cookies. It also promotes looking at and comparing different recipes for the cookies.

Counting, Earning, and Spending Money

There are many books in which money plays an important role. Some books have a story line based on earning or spending—for example, ***Arthur's Funny Money*** (Hoban 1981) and ***Alexander Who Used to Be Rich Last Sunday*** (Viorst 1978). Classroom activities based on these and similar books are described in chapter 7. Other books, such as the one below, involve saving a large number of coins in a piggy bank.

A Chair for My Mother (Grades 1–3). A slowly collected jar of loose change, representing a family's savings, plays a prominent role in this book by Vera B. Williams (1982). The change is eventually spent on a comfortable chair for the mother who works all day on her feet in a diner. The image of the jar of coins suggests estimating and counting coins to find the value of a classroom jar of coins. Vera Williams's books are also notable for the intricately patterned borders around the pages.

Origami

The Japanese art of paper folding uses a wealth of geometric ideas. In describing the most basic constructions, language related to fractions, midpoints, angles, line segments, shapes, and so on, will arise naturally. There are many how-to books on origami. In some, the diagrams are incomprehensible to many readers. The best way to learn how to fold a certain pattern is to follow step by step as it is performed by someone who knows how. Such an expert can often be found among students in upper elementary school classes. Below are two stories related to origami and a resource book. The two recommended stories both involve paper cranes, which are somewhat difficult to fold, but you can find simpler alternatives in the resource book.

The Paper Crane. In this story by Molly Bang (1985), a family that owns a restaurant helps an old man, who rewards them by folding a paper crane from a napkin. The bird comes to life and brings them good fortune.

Sadako and the Thousand Paper Cranes. This book by Eleanor Coerr (1977) recounts the story of a girl afflicted by radiation sickness from the bomb at Hiroshima and how she folded paper cranes from the paper that was used to wrap her medicine. Her hope was that if she folded 1000 paper cranes, her prayers would be answered. Subsequently, the paper crane became an international symbol for peace. A curriculum unit for grades 4–6 based on this book is part of a Literature/Social Studies Program (Sundance Publishers and Distributors, 1990). Mathematics also occurs in this unit in the consideration of time lines and maps.

Complete Origami. This book by Eric Kenneway (1987) has directions for projects at a wide variety of levels as well as fascinating background information on origami. To follow up on the Molly Bang book, it would be appropriate to teach children how to make a flapping bird (under "Action Models").

Patchwork

Many traditional patchwork patterns afford a context for geometry investigations involving fractional parts, congruence and similarity, naming and finding shapes, measurement of length, angle, and area, and tessellations. The children's books below do not explore these geometric aspects of quilts—they involve the personal and social implications of patterns made from a large number of

BEST COPY AVAILABLE

patterned scraps—but they might suggest showing other quilt patterns and exploring their geometric properties. The resource books below (by Gutcheon and Beyer) are wonderful sources of such patterns.

Children's stories

The Quilt, by Ann Jonas (1984)

The Patchwork Quilt, by Valerie Flournoy (1985)

The Quilt Story, by Tony Johnston (1985)

The Patchwork Cat, by Nicola Bayley (1981)

Resource books

The Perfect Patchwork Primer, by Beth Gutcheon (1973)

Patchwork Patterns, by Jinny Beyer (1979)

Songs

Traditional folk songs are one of the most attractive ways for young children to learn rote counting, both forward and backward, as well as sequence and patterning. Some songs have been presented as books, where the auditory aspects of the songs are supplemented by artwork. Two examples are described below. Two others, ***Ten Bears in My Bed*** (Mack 1974) and ***The Twelve Days of Christmas*** (Kent 1971), are described in chapters 2 and 4, respectively.

Over in the Meadow (Grades Pre-K–1). The lovely illustrations in this book by Ezra Jack Keats (1971) reinforce a classic counting song. Each verse involves an animal mother with a different number of young. A follow-up activity would be to look at pictures of animals and their babies and to classify them by the number of young.

I Know an Old Lady Who Swallowed a Fly. She then swallowed a spider to catch the fly, and a bird to catch the spider.... The song in this book by Nadine Bernard Wescott (1981) suggests a follow-up activity of ordering a series of pictures of animals by size.

Mysteries

A mystery that is solved by piecing together clues can be used to encourage logical reasoning activities. Present students with a collection of clues about a mystery number or a mystery shape. If the clues are shown one by one, students can be challenged to decide at what point they can be absolutely sure about the identity of the mystery number or shape. They can also consider what clues are superfluous because they duplicate information that is already known. No particular examples of this type of children's book are given because almost any mystery with a series of clues can be used to promote such an activity.

BEST COPY AVAILABLE

Activity Sheet 1.1

Sort your pennies.
Arrange them on this grid to make a graph.
Label your graph.

2

INTRODUCE MANIPULATIVES

Manipulatives play an important role in the learning of mathematics. One of the basic assumptions underlying the NCTM standards for grades K–4 in *Curriculum and Evaluation Standards for School Mathematics* (1989) is that "the curriculum should actively involve children in doing mathematics," which implies that teachers must "make extensive and thoughtful use of physical materials to foster the learning of abstract ideas" (p. 17). The standards for grades 5–8 also assume that "every classroom will be equipped with ample sets of manipulative materials and supplies" (p. 67). Books can provide motivation for students' meaningful use of manipulative materials in many ways. Sometimes these uses go beyond the context of the book.

There are two general types of manipulative materials suggested in this chapter. The first type is counters. Counting books dominate lists of children's books related to mathematics. Often counting books are explicit in their use of counting. Many lack a context or a story line and in fact seem dull to children. Look for counting books that have an inherently attractive story or theme and that require some interaction (for example, predicting what is on the next page or searching for a number of hidden objects). Sometimes pictures in a counting book can be copied and used as a context for children to make up their own number stories. An example here is *Ten Apples Up on Top* (LeSieg 1961).

Sometimes counters are not used explicitly but play an important role in the plot, as the caps do in *Caps for Sale* (Slobodkina 1984). Here there is the added advantage of a plot that provides a context for copying activities, which are very useful to the primary teacher for the assessment of number-related skills. Another book of this type is *More Spaghetti, I Say* (Gelman 1977),which here is used to stimulate the use of one material—mixed dried pasta—in a great number of ways! This particular material has proved especially useful because it is so inexpensive, so available, and shows up well on an overhead projector. Since a very small amount is used as the manipulative, very little is wasted. (Should the issue of wasted food be of concern to you, most activities can be interpreted with other materials, such as mixed nuts, as shown in the "Other Books" section at the end of this chapter.)

Any book that describes varied attributes of something can be used to make an attribute set, which can be used for many classification and reasoning activities. The example here of a cat attribute set can be inspired by *Millions of Cats* (Gag 1977) or by *So Many Cats* (de Regniers 1985). Additional books that suggest similar creation of a manipulative are suggested in the "Other Books" section.

One technique used in this chapter that can be applied in many other situations is the "little fold-up book" suggested for *Caps for Sale*. The experience of folding such a book can suggest thinking about how books are made in general; pages are printed on both sides of a large sheet and folded before being cut. Books made this way would have pages grouped in certain numbers (four pages in a small fold-up book, eight if it was folded again, sixteen if folded again,...). Children might design their own books of this type and figure out how the small pages must be laid out.

Ten Apples Up on Top

Theo LeSieg (1961)

Summary of the Story

In this vivid and amusing counting book, three animals compete to have the most apples on top of their heads. Each starts with one, then has two, and so forth up to ten. Then the animals are chased by a variety of other animals, all of whom seem to want to take away the "ten apples up on top." Eventually, everyone crashes into a huge apple cart, and after a page of chaos, everyone has ten apples up on top.

Classroom Sketch (Grades 1–2)

Grouping by tens
Place value

Each group of two to four children will need a mat (one-half of activity sheet 2.1), a box or basket (e.g., a basket from fruit sold in a supermarket), and twenty to fifty small counters to represent apples (bingo chips or buttons or large dried lima beans spray-painted red).

1. Read the story to the children. Show a model of three animals with a number of chips on their heads (say 8, 11, and 10). You might sketch the animals on the chalkboard and use red apple shapes attached with self-adhesive magnetic tape (if your board is suitably backed) or with masking tape. You could also use a transparency of activity sheet 2.1 with red bingo chips on an overhead projector. Which animal has ten apples up on top? What must be added or taken away from each to make ten? Add or take away the extra chips. When all animals have ten, how many chips are there on one animal? Two animals? Three animals?
2. Give each group of two to four children a mat with pictures of animals and a small handful of twenty to fifty chips (red, if possible) in a box—the apple cart. First ask the groups to guess how many chips they have, then check their guesses by putting ten apples on top of each animal, leaving the leftovers in the apple cart.
3. Use the chips and mats to pose problems involving counting on and counting back from two-digit numbers. In each group, have children put ten apples on two of the animals and eight on another. How many apples have been used? Now put on three more apples. How many are there now? Now take off two. How many now? Eventually, encourage children to try to visualize these problems and answer them in their heads before checking the answers with the materials.
4. Explain how to use the mats for an exchange game (see below). Children can later play this game in small groups.

Follow-up: Exchange Game

Each child in a group of two to four should have a board made from half the animal sheet (five animals). Cut the apple grid (activity sheet 2.2) into ten-strips and loose apples. Children toss two dice and take that many loose apples, which they arrange on the animals' heads (fig. 2.1). Ten apples can be exchanged for a ten-strip. The object is to fill the board. If desired, have children say after each turn how many apples they have, for example, "I have three ten-strips and four apples, so I have thirty-four apples."

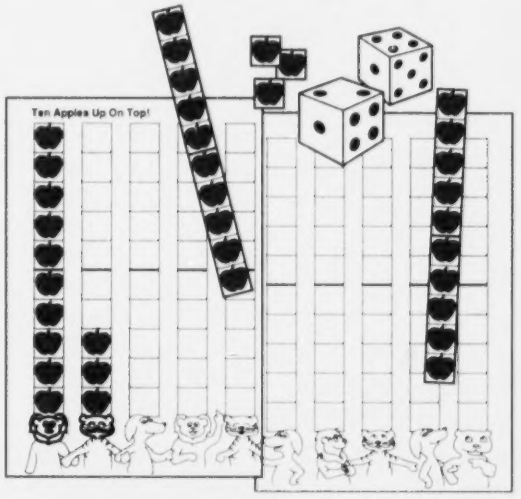
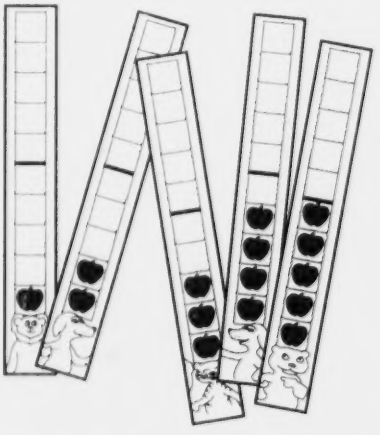


Fig. 2.1

Follow-up: Other Uses (Grades K–2)

For the following activities, activity sheet 2.1 can be divided in various ways. For apples you can use the chips described above or apple strips made by gluing the apple grid to cardboard, coloring it, and cutting it into strips. Some activities that use these materials are given below:

Seriating pictured sets of 1–10. Glue the animal sheet (activity sheet 2.1) to cardboard, color in one to ten apples on each head, and then cut vertically into strips. Use these strips to act out the story in the book or have children arrange them in order. Children can also match the strips to numerals.



Matching sets to numerals. Write numerals on each animal, perhaps counting up or counting down. Have children arrange apple counters accordingly, use apple strips to match the numeral (cut the grid into strips of one to ten apples), or color in apples above each animal on a worksheet (fig. 2.2).

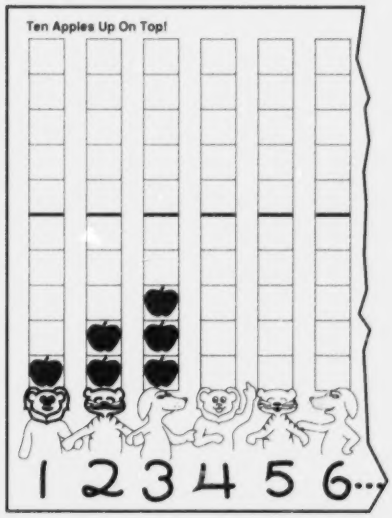


Fig. 2.2

Describing sums to a given number. Have children toss a die and put the number of apples indicated on top of an animal; then describe what they see. For example, if they roll a four, they would say, "I have four full spaces and six empty spaces, making ten spaces in all," or write,

"4 + 6 = 10."

If you would rather have the children work on sums less than ten, cut the animal sheet horizontally to get the number you want (fig. 2.3). For example, if it is cut at six spaces, to match the numbers on most dice, the students would be forming all sums of six, such as 2 + 4 = 6.

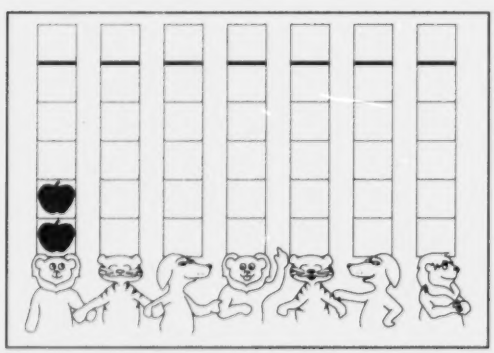


Fig. 2.3

Children can record all the different sums they can make in a little "apple" book (fig. 2.4).

Matching numerals to sets and spaces (a game). Each child should have a half-board from activity sheet 2.1 with five animals and apple strips with one

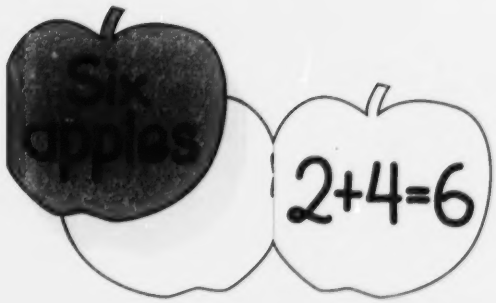


Fig. 2.4

to six apples (fig. 2.5). The children take turns tossing a die, choosing a corresponding number strip, and putting it on one of their animal heads. The object is to fill the board. Once a strip is placed, it cannot be moved. Children can take two strips on a turn if desired, that is, if they roll a 6, they can take a 2-strip and a 4-strip.

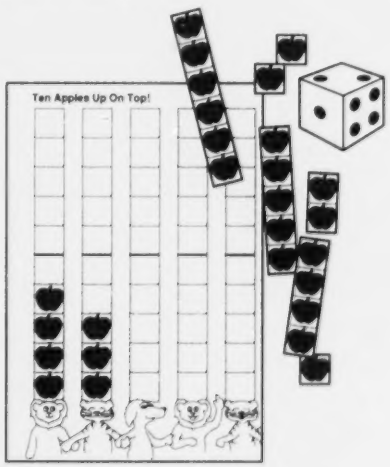


Fig 2.5

Acting out sums of two numbers. For this activity, cut activity sheet 2.1 into five pieces, each having two animals on it. To add two numbers, children can either arrange beans in the spaces or use apple strips cut from the apple grid (fig. 2.6). Note here how the "five structure" (the heavy line across the animal sheets dividing the strips into two groups of five) helps children to recognize sums such as 7 + 8 as two fives and a two and a three, or as a ten and a five, or as fifteen).

Fig. 2.6

Making a graph. Have three kinds of apples (perhaps Golden Delicious, Red Delicious, and Granny Smith) and cut out three of the animal strips. Label the animal strips with the types of apples. Have each child taste each apple and color in a space over the animal that corresponds to the apple he or she likes best (fig. 2.7). *Which apple was the favorite?*

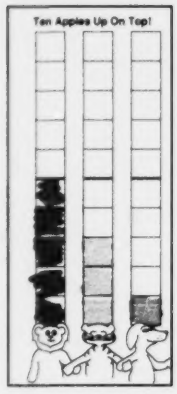


Fig. 2.7

Caps for Sale

Esphyr Slobodkina (1984)

Summary of the Story

A peddler sells caps of different colors that he carries neatly stacked on top of his head. He stops to take a nap against a tree and awakes to find that all his caps except one are gone; they have been taken by monkeys up in the tree. In a fury, he tells the monkeys to give back his caps, shaking his fist and stamping his feet. But the monkeys only copy his actions. Finally, in despair, he throws his own cap on the ground. Of course, all the monkeys do the same thing, and he gathers up his caps.

Classroom Sketch (Grades K–2)

Patterning
Counting
Addition
Subtraction

For this activity, children will need a manipulative. Copy the mat on activity sheet 2.3 to make peddler mats and monkey mats. (For small-group work, you will need one peddler mat for each four to five monkey mats.) Caps are easily made by spray painting large dried lima beans in different colors—for example, red, black, orange, and white (the natural color of the beans).

1. Read the story to the children. If desired, let the children enact the story; one child is the peddler and the rest are monkeys. At this stage they can use imaginary hats.
2. Introduce the manipulative. Have one child arrange the "hats" (painted lima beans) on a peddler mat as they are in the story—first the peddler's own hat, then four of one color, four of another color, and so on. Then have the children close their eyes, while you or a child (the "monkey") takes a hat and hides it. Have children open their eyes and try to figure out which hat was taken. If this is easy for the children, repeat, but mix up the colors of the hats, or start with five of each color. Children can later play this as a game in small groups.
3. Give each child a monkey mat and have one child (or yourself) be the peddler. Whatever the peddler does, the monkeys must copy. This can also be done in small groups. Note that this activity is very useful for informally assessing children's number skills. Depending on the children's interests or level, the peddler can do several things with the hats:
   - Arrange the caps in a pattern by color—the monkeys must continue the pattern.
   - Lay out a specific combination of colors—three whites and four reds, for example.
   - Give only verbal clues —"I have five hats—all are black or red, one is red," for example.

Follow-up: Making a Fold-up Book

If the two activity sheets on the pages after activity sheet 2.3 are reproduced back-to-back exactly as they are printed here, children can fold them to make a little book. You might want to let the children figure out themselves how to fold the pages so that the book pages are in order, or you might want to give explicit directions: *Place the page so that you can read the book title. Fold on the line of dots so that the title is on the outside. Then fold on the line of dashes so that the title is still on the outside. Staple twice along the line of dashes. Cut off just enough paper along the top fold so that you can open the pages.*

Note: For more ideas on using this book to teach mathematics, see "Mathematics from Children's Literature" by Rosamond Welchman Tischler (1988, pp. 42–47).

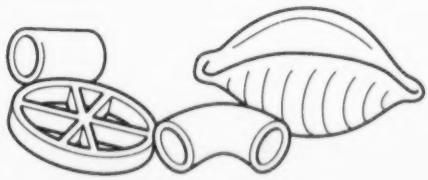
More Spaghetti, I Say!

Rita Golden Gelman (1977)
Illustrated by Jack Kent

Summary of the Story

In this Read It Yourself book, one monkey asks his friend to play but is refused because the friend is too

involved with spaghetti. In verse, the friend enumerates a huge variety of things that can be done with spaghetti. Finally the monkey tries it and finds it wonderful, and then the tables are turned—he won't play when his friend wants to play because he is too busy with spaghetti!



Classroom Sketch (Grades K–3)

Counting
Patterning
Graphing
Addition
Comparison of number
Shopping
Measurement
Place value

In using this book to inspire the following activities, spaghetti is interpreted rather freely as pasta. Pasta is available in a wide variety of shapes and suggests a number of mathematics-related activities. It can be a unifying theme in a mathematics learning center. Below is an introductory lesson, leading to children's independent or small-group use of the material in a learning center. Only a small amount of pasta is required. A culminating activity might be to make a pasta soup (using clean pasta instead of the pieces that have been handled) or to throw some of the pasta into a "stone soup" (see chapter 1). The suggested learning-center activities assume that you have a container with a mixture of wagon wheels, macaroni, ditali, and shells. These show up well on an overhead projector. You can of course adapt the ideas for other types of pasta or other mixtures of materials. Many activities also require a small scoop (such as for coffee), cards with the outline of a cooking pot, and copies of activity sheets 2.4 to 2.7.

1. Read the book to the children. Show them a scoopful of the mixed pasta. Ask them what they could do with it. Encourage the children to handle and arrange the pasta. Probably some will suggest sorting it into the different types, or making patterns with it, or counting it. If these ideas do not emerge, elicit them with questions such as, "Are all the pieces the same?" "How could you arrange them?" "How many wagon wheels do you think there are?" "How many shells?" "How many in all?"

2. Explain that some pasta, like that in the scoop, will be available in a learning center and that there are several activities they can do with it. Select from the ideas below some that are appropriate for your students. Demonstrate how each activity is to be done. Have a chart with each child's name and places for the children to check off which pasta activities they have done. You might finish the introduction by reciting the following adaptation of the book:

> You can *sort* all this pasta,
> Then *graph* it, you see!
> *Compare* it, and *add* it,
> Come *shopping* with me!
> You can *estimate*, *count* it,
> And *measure* with glee—
> Learning centers with pasta
> Are fun, you'll agree.

Follow-up: Learning Center Activities

Sorting. Children take a small scoop of pasta and some pot cards. They sort the pasta into pots so that each pot has only one type of pasta (fig. 2.8)

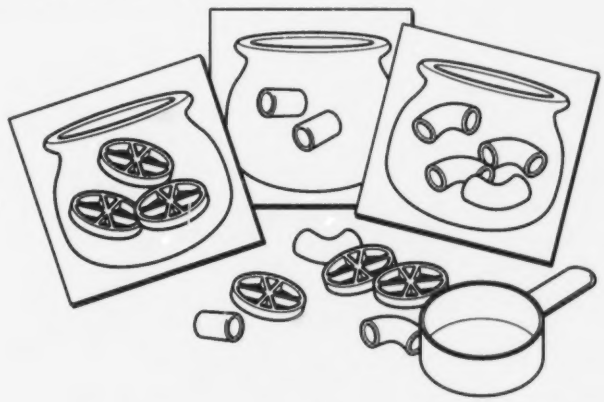


Fig. 2.8

Sorting—extended. Color some pasta with food color or include some that is naturally colored. Children can then sort by more than one attribute, possibly using yarn loops to make a Venn diagram.

Patterning. Encourage children to make patterns with the pasta. To get them started, have a collection of pattern strips for them to extend. (See the top of activity sheet 2.5 for samples.) Children might classify the patterns, recognizing that some are alike (i.e., an A B A B A B ... pattern or an A B B A B B A B B ... pattern).

Counting. Have children put a specified number of pieces of pasta (perhaps zero to ten) in each pot on some pot cards. They can match number to numeral by placing little numeral cards on the pot cards (fig. 2.9). When this is done, you can play a quick game. Have children close their eyes while you (or another child) move one piece of pasta. Challenge the children to figure out what was done.

Counting—extended. Have the children estimate and count how many of each type of pasta fit in a little scoop. What do they notice? (Probably they will say that fewer of the larger pieces fit in the scoop, and more of the smaller pieces fit.)

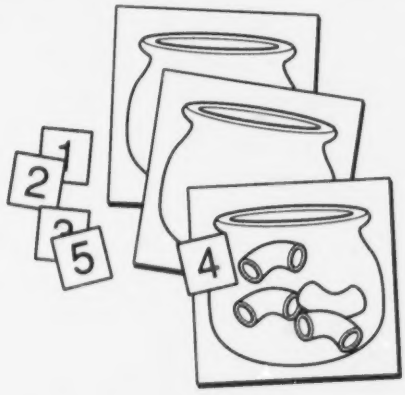


Fig. 2.9

Counting—a game. This game resembles bingo. Each child has a "Pasta Soup" recipe card to complete. (Activity sheet 2.5 has two samples. Adapt one of these and make some more with different numbers.) Children take turns rolling a die and taking as many pieces of pasta as the number rolled—however, on each turn all the pieces taken must be of the same type (fig. 2.10). The first player to complete the recipe card wins. (If desired, children could finished the activity by actually combining their ingredients and cooking them!)

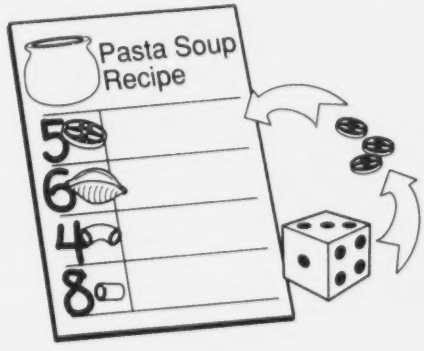


Fig. 2.10

Graphing. Children take a small scoop of pasta and a graphing sheet (on activity sheet 2.5). They sort their pasta and arrange it on the grid. Then as they remove the pieces of pasta, they shade in the corresponding squares, making a bar graph (fig. 2.11)

Graphing—extended. Compare the graphs made by several children. *How are the graphs alike? How are they different?*

BEST COPY AVAILABLE

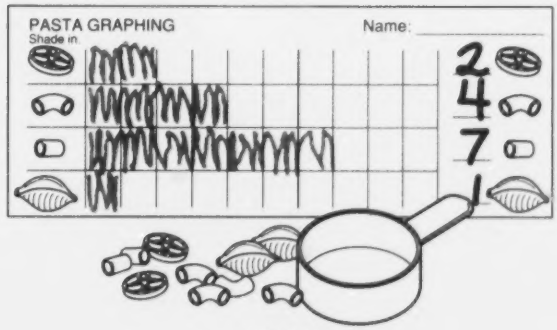


Fig. 2.11

Addition—pasta pots. Children take a small scoop of pasta and the addition activity page on the bottom of activity sheet 2.4. They sort their pasta and place different types in the large spoons on the activity page, writing how many in the corresponding small spoons. They then push the pasta down into the large pot and record the total in the corresponding small pot. The record of this action looks like the standard addition format. Note that each spoon gives six possible combinations of two types of pasta; so there are six spaces on the activity page. Children can then be asked to write the addition sentences in horizontal form at the bottom of each example, as shown in figure 2.12.

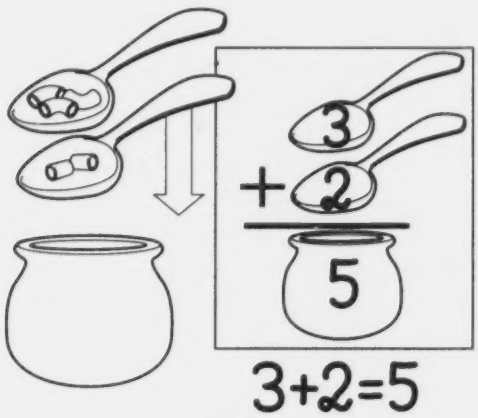


Fig. 2.12

Addition—extended. Encourage children to "count on" by modifying this format. Place a numeral on a pot to indicate its contents. Children put pasta in one spoon, push the pasta into the pot, and figure out the new total number (fig. 2.13).

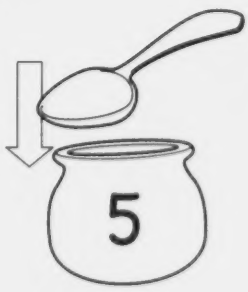


Fig. 2.13

Comparison—Pastaman. Children will need a cut-out "pastaman" shape, the size of the dotted circle on activity sheet 2.6. For class demonstration, you can use a yellow lid from a coffee can taped to a Popsicle stick. Tell the children that Pastaman always wants to eat the contents of the pot with the greater number of pieces of pasta. Notice that if he is placed facing the pot that he wants to eat, his mouth forms the inequality symbol. When the amounts in the pots are the same, he can't make up his mind, and his mouth looks like an equals sign! (See fig. 2.14.) You will need a second coffee can lid for the equals sign. To perform this activity, children take a small scoop of pasta, sort the pasta, and put two types of pasta in the pots at the top of activity sheet 2.6. They place the cut-out Pastaman in the dotted circle to show which pot he wants, then record the numbers and the inequality symbol in the corresponding space below. (When faced with an example such as that of two large shells being compared with three small ditali, some children might say that Pastaman would prefer the shells because there is a greater volume of pasta in them. Help children see that this would be correct if Pastaman cared about volume, but in fact he likes to count and always wants the pot with the greater number.)

Fig 2.14

Shopping. For this activity at the pasta store, children need some sort of money, real or play coins

to model the shopping list (fig. 2.15). Activity sheet 2.7 has a price list and six examples of shopping questions that vary in difficulty level and type. (Note that the first question is routine; the second has many answers; the third involves two steps; the fourth has more than one answer; the fifth requires a lot of thinking, and one challenge—to buy six pieces for five cents— cannot be done; and the sixth suggests an interesting creative challenge for children—to design their own picture.) The two spaces at the bottom allow you to write some problems of your own. Questions can be color coded by difficulty level. You might provide a few sample questions; then give children blank cards so that they can add to the collection of questions themselves. The questions can also be made more challenging by changing the price list.

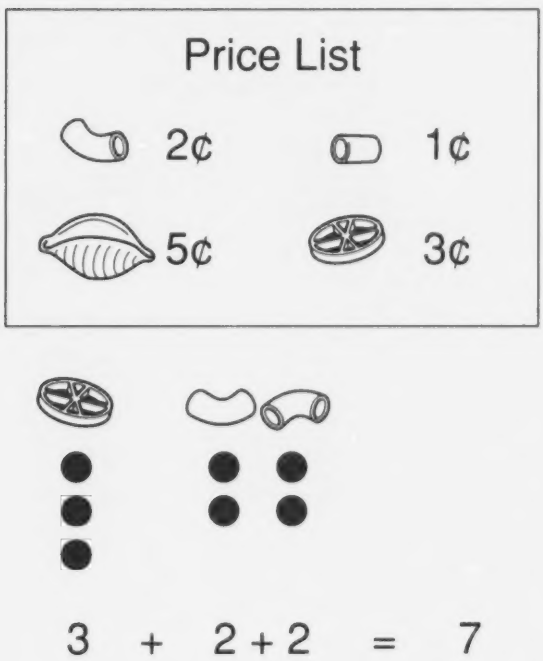


Fig. 2.15

Estimation and place value. Fill a clear container with one type of pasta and challenge children to estimate the number of pieces in the container. When all children have had a chance to make an estimate, have some children count the pasta using egg cartons (fig. 2.16). They can put 10 pieces in each cup, and when they are finished, they can see how many tens they have and how many ones are left over. If the children are counting numbers over 100, cut two cups

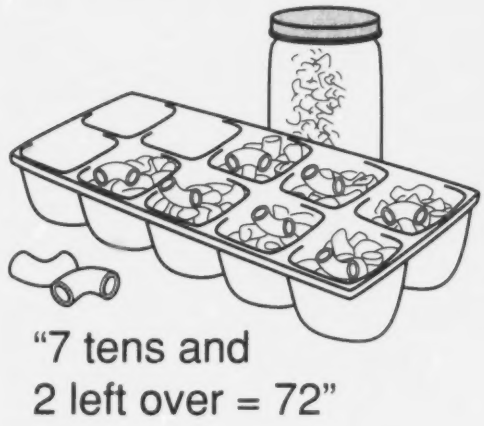


Fig 2.16

off a carton for one dozen eggs leaving ten cups, so that 100 pieces will fit in each carton.

Measuring—length. Pasta can be used as a unit of length. Ask children to find out how long a crayon is. They can measure it with wagon wheels, then measure it with macaroni (fig. 2.17). *Which is better to measure with? Why?*

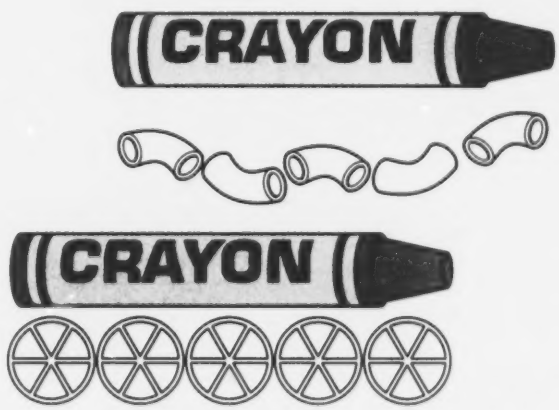


Fig. 2.17

Measuring—weight. Use a two-pan balance (see fig. 2.18). Ask children to guess which weighs more, ten wagon wheels or ten macaroni. Try other combinations. *Which weighs more, a scoopful of wagon wheels or a scoopful of macaroni?*

Pasta can also be used as a unit of weight. Have children pick one type of pasta and try to balance small objects with it. Make a chart as in figure 2.18.

If you are cooking pasta, you might weigh it before and after cooking and note the change.

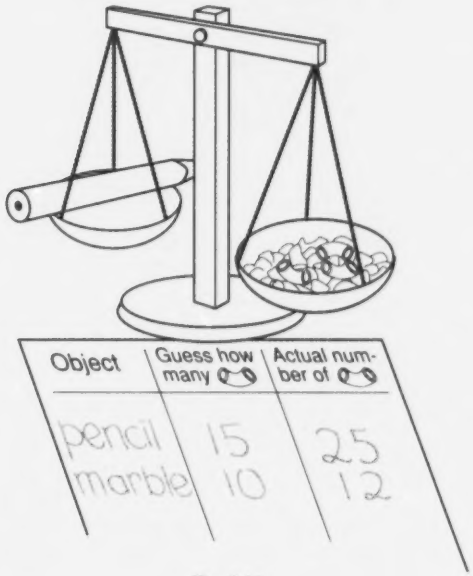


Fig. 2.18

Millions of Cats

Wanda Gag (1977)

Summary of the Story

An old man searching for a kitten finds "hundreds of cats, thousands of cats, millions and billions and trillions of cats!" He has difficulty choosing just one; so he brings them all home. Eventually all the cats except one scrawny kitten get into a terrible fight and destroy each other. The man and his wife are happy with the remaining kitten, who now seems the most beautiful of all. Issues are raised about how much food and drink a huge number of cats would consume.

The book *So Many Cats* (de Regniers 1985) could also be used in this sketch. (See chapter 6 for a description of this book.) Both of these books describe a variety of cats in loving and individual detail.

Classroom Sketch A (Grades K–2)

Classification
Graphing

1. Ask children to search for pictures of cats, either at home or in school. (There are many nice glossy pictures in pet-food advertisements.)
2. Spread out all the pictures that children have collected and ask the children to classify them. Suggest sorting into loops. If the method of classification requires overlapping loops, it could be represented in a Venn diagram. For an additional challenge, ask a child to arrange the pictures by a secret rule, without labeling the loops in the Venn diagram, and challenge other children to guess the rule.
3. Probably at least one method of classification will separate the cat pictures into nonoverlapping loops (e.g., by color or markings). Suggest that this method can also be represented on a graph.

Follow-up: Estimation Challenge

Display a large number of cat pictures—too many to count by eye—on a poster and ask the children to estimate how many there are. When all children have had a chance to estimate, put the poster flat on a table and have children put a marker on each cat. Count the markers to check the estimates.

As an additional challenge, ask children to figure the total number of eyes or legs possessed by all the cats on the poster.

Follow-up: Surveys about Pets

Children can conduct various pet-related surveys. *What pet would you most like to have? What pets do you have? How many pets do you have?* Children can collect data and find a way to represent them graphically. The results of their survey might be used to make estimates, for example, of how many cats are owned by people who go to their school or how many pets live on their block. Many pet-related books could support this investigation.

Classroom Sketch B (Grades K–6)

Classification
Logical reasoning

Make an *attribute set* by mounting the pictures from activity sheets 2.8 and 2.9 on cardboard. This set of twenty-four pictures represents cats in all possible combinations of size (large and small), poses (lying, sitting, standing, and jumping), and markings (black, white, and striped). The pictures on the activity sheets were made by a thirteen-year-old who first drew the cats in four positions, made two photocopies, and colored the copies in two different ways. When the twelve resulting cat pictures were finished, they were reduced on a photocopier to produce the smaller set. This procedure could be followed by students in your

classes to make their own attribute sets.This attribute set can be used for many classification activities, including the following:

Which one is missing? Give children the set of cat cards after taking one card out. Challenge the children to find which cat is missing. Encourage children to share their strategies. Most will classify the cats in some way and count the piles. Children can play this as a game in small groups.

Guess my rule. Start sorting the cat cards into two or more groups. After a few pieces are placed, challenge children to tell where the next pieces should be placed. Allow all children to try, correcting their placement if necessary. Eventually, have a child describe the rule. Children can play this as a game in small groups.

Classification in Venn diagrams. Use two or more loops of yarn and movable labels to arrange the cat cards. Start with labels that describe nonoverlapping sets, such as *large* and *small* or *white, striped,* and *black.* Later, when children understand how the loops represent a set, try a pair of labels such as *small* and *black.* Challenge children to figure out what to do about the cards that belong in both loops—probably someone will suggest moving the loops so that they overlap. When children understand the interpretation of the overlapping loops, try a guess-my-labels activity. Place a few pieces in the loops, challenge the children to place some more, and eventually explain what the labels could be. Ask the children to describe the pieces that go in the four areas of the diagram using the words *and* and *or* with the attributes.

One-difference trains. Have children analyze the number of ways in which two cards are different—some combination of size, position, and markings. Once they know what it means for two cards to differ in just one way, they can construct a *one-difference train*—a row of cards where each card differs from its neighbor in just one way. This can be an individual or a cooperative activity, where children try to use all the cards in a one-difference train. An extra challenge is to make the train close up on itself. If children wish to play this as a competitive game, they should first share the cards evenly, then take turns adding to either end of a one-difference train. A child who cannot place a card is out of the game. The last player to stay in the game is the winner. Children can also try to construct two-difference trains.

Difference puzzles. The concept behind one- and two-difference trains can be used to construct puzzles. In fact, children can easily construct these puzzles themselves. Have them select a few cards, place them on paper, and trace the outlines, as shown in figure 2.19. They should then draw lines on the paper between some pairs of cards to indicate the number of differences between them—a single line for one difference, a double line for two differences, and a triple line for three differences. When the cards are removed, the puzzle is to find the way to put the cards back on the outlines so that the lines correctly indicate the number of differences.

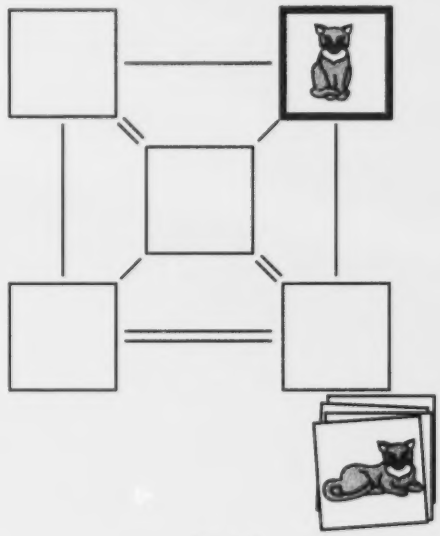


Fig. 2.19

Other Books

Attribute Sets

Many books suggest the construction of an attribute set that can be used for many activities, as was done for *Millions of Cats.* Any book that mentions many variations of a general type of object is suitable. Here are some suggestions:

Ruth's Bake Shop. There are pages and pages of delectable cookies that vary in shape, color, and texture in this book by Kate Spohn (1990).

Clementine's Winter Wardrobe. Another book by Kate Spohn (1989) shows amazing variations in articles of clothing.

Both books by Spohn suggest not only the formation of sets of images to sort but also estimation and counting.

All the Pretty Horses. This book by Susan Jeffers (1985) contains lovely images of horses of many different kinds.

One Fish, Two Fish, Red Fish, Blue Fish. Images can be drawn from this book by Dr. Seuss (1987) to count and sort. Children might draw some fish of their own with specific attributes (see fig. 2.20).

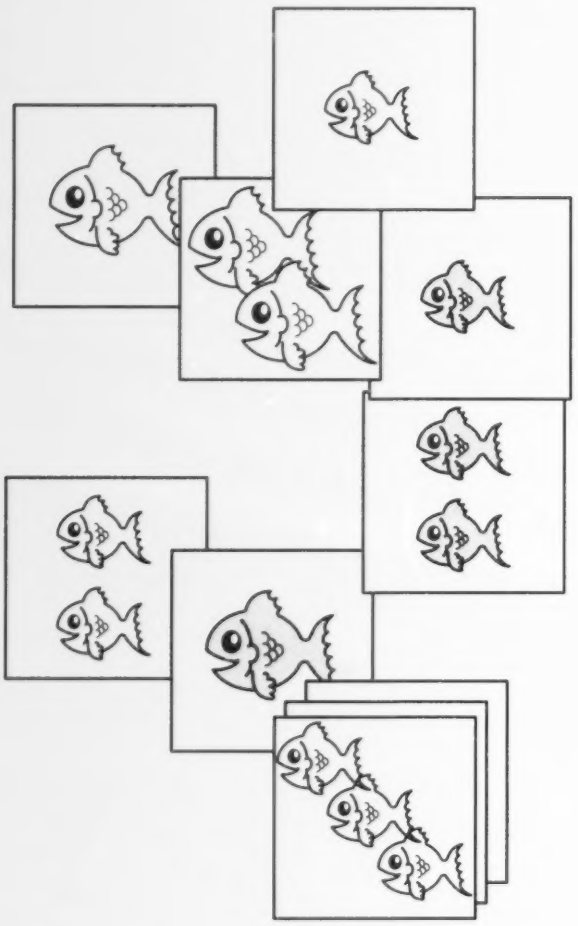


Fig. 2.20

Manipulatives for Number

The themes of many books, both counting books and others, can be used to develop a manipulative for number. The examples below are listed under the manipulative they suggest:

Teddy bears

Ten Bears in My Bed (Grades K–2) This counting-down book by Stanley Mack (1974) is based on the old song, "There were ten in the bed, and the little one said, 'Roll over! Roll over!' So they all rolled over and one fell out. There were nine in the bed and the little one said...." You can use the commercial plastic teddy bear counters, but it is also very natural to act out this book with the tiny graham cracker bears, and let the bears be eaten as they fall out of bed! To extend the activity, do not allow eating until the end and vary the song. You can insert such phrases as, "So they all rolled over and two fell out," or "... three climbed in." Children can tell stories about the bears in the bed themselves and enjoy a relatively nutritious snack. Note that the bears come in two positions and three different colors (see fig. 2.21); so they can be used for sorting, too.

Fig. 2.21

Fish

Swimmy (Grades 1–3). This book by Leo Lionni (1987) is another book that can be related to an edible manipulative and depends on the idea of large numbers. It relates the story of a tiny fish (Swimmy) who discovers how he and many other similarly small fish can protect themselves from the huge fish who try to eat them—they form themselves into the shape of an even larger fish that frightens the predators away. Small crackers in the shape of fish are available and can be used as a manipulative (fig. 2.22). The story concerns finding protection in large numbers; so the crackers can be used in an estimation-and-counting activity. Spread a bag of the crackers in a fishlike shape on a large sheet of clean paper and ask students to estimate the number of crackers. Then each group

of children can be given a scoop of the crackers to count. They can then add the numbers from the groups to find the total. The fish crackers can also be used to act out "fish" stories as described above for the bear crackers.

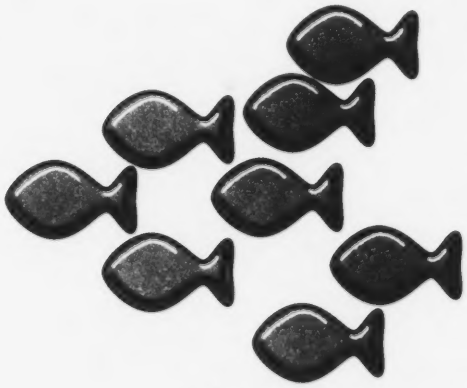
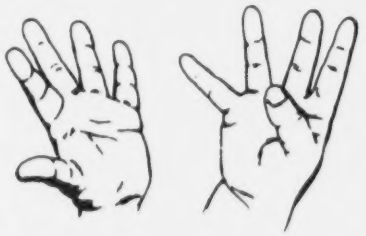


Fig. 2.22

If you would like to make a more durable manipulative for this activity, see *Workjobs II, Number Activities for Early Childhood* by Mary Baratta-Lorton (1979), which gives directions for making little fish from spray-painted dried lima beans and scraps of felt.

Fingers



Count on Your Fingers African Style (Grades K–2). This counting book by Claudia Zaslavsky (1980), illustrated by Jerry Pinkney, explicitly describes the most readily available manipulative of all, our fingers, and looks at the ways that different African cultures count and represent number on their fingers. It develops understanding of different ways in which numbers can be thought of as sums and encourages children to make up new finger signs for numbers based on arithmetic relationships.

Trains

Books about trains can establish a context for a versatile number material. The cars of a train are hitched together, which suggests putting numerals on them and ordering them. Freight is loaded into cars, which suggests putting a manipulative (wooden blocks, chips) into freight cars. Among the many books for young children about trains that suggest the use of such a manipulative are the following:

The Little Red Caboose (Grades Pre-K–1). This book by Marian Potter (1978), with pictures by Tibor Gergeley, emphasizes the sequence of train carriages lined up behind an engine—"And the little red caboose always came last!" Thus, it is suitable for establishing the order of cars labeled with numerals (fig. 2.23).

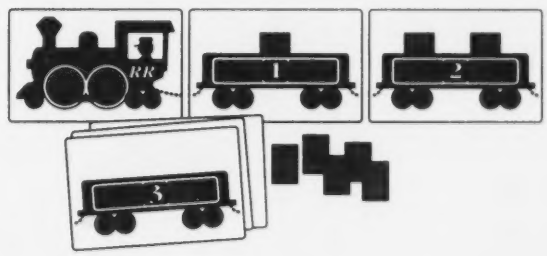


Fig. 2.23

Freight Train. The illustrations in this book by Donald Crews (1978) are wonderfully vivid and compelling.

BEST COPY AVAILABLE

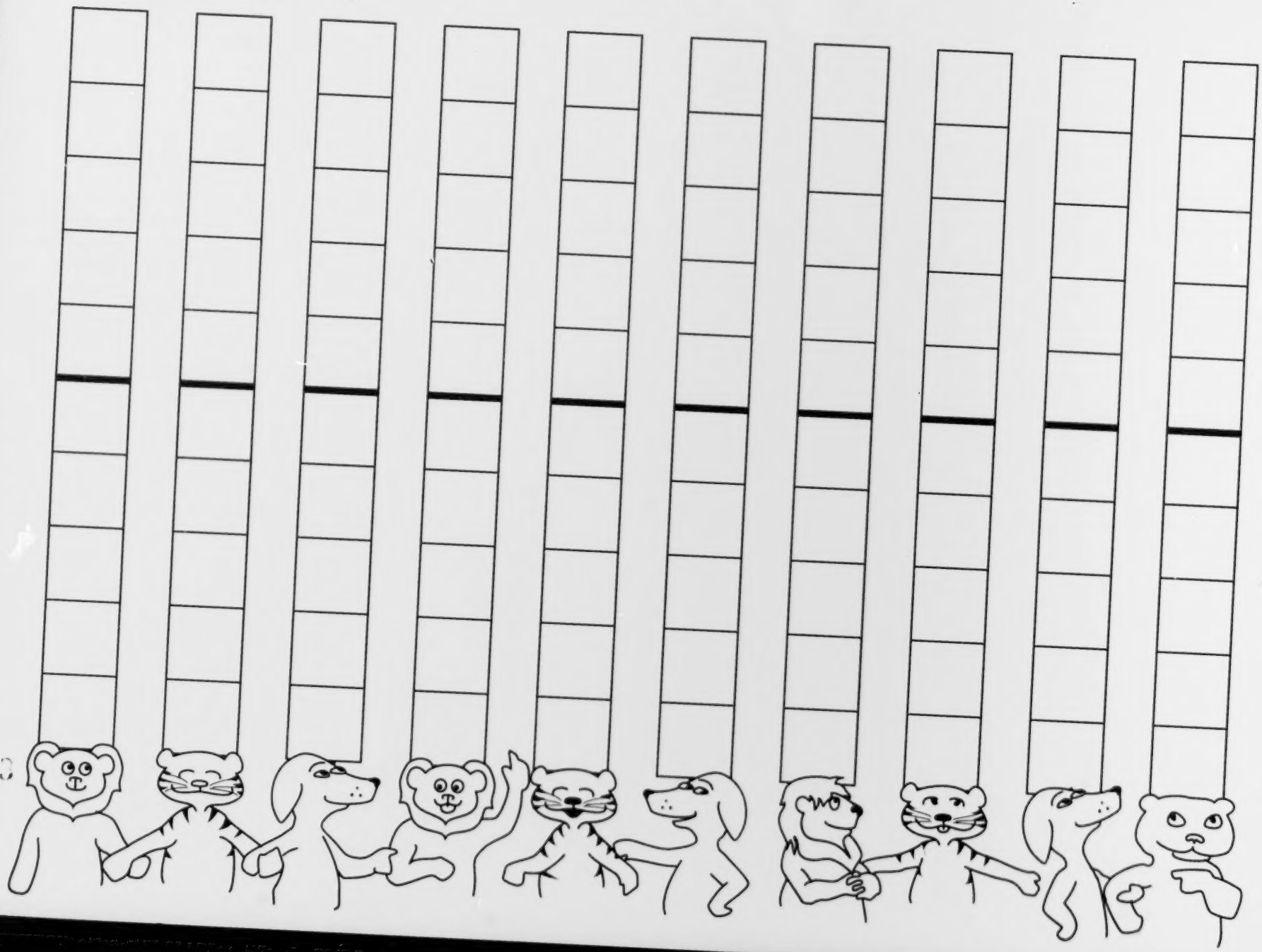
Mixed Nuts

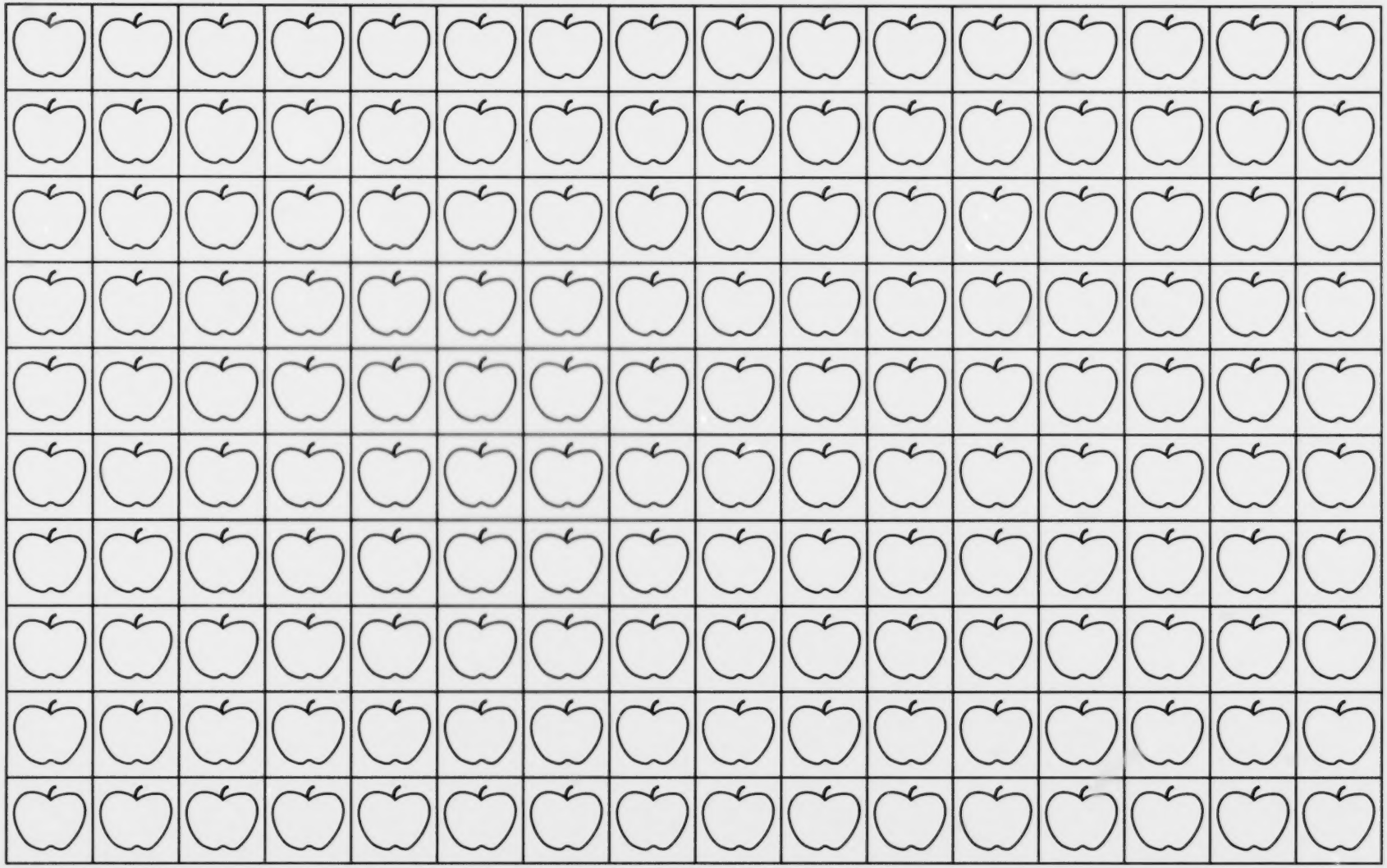
Many ideas developed for pasta (*More Spaghetti, I Say!)* can be used with other materials. In the fall, a class might go on a walk in a nearby park and collect an assortment of at least four different kinds of nuts, acorns, or large seeds. Or children might use a bag of mixed nuts in their shells from the supermarket. One advantage of this material is that handling the nuts in their shells poses no health risk if children later open and eat them. To encourage the use of these manipulatives, use a book in which squirrels collect nuts for later consumption, such as the following:

The Tale of Squirrel Nutkin. This classic by Beatrice Potter (1987) was first published in 1903. Squirrels are gathering nuts for the winter.

The Meanest Squirrel I Ever Met. A small squirrel in this book by Gene Zion (1982) plays all sorts of games ("nuts in rows," "nuts in circles," "nuts in little piles") with the pecans, walnuts, and hazelnuts that are to be his family's Thanksgiving dinner. A mean squirrel cheats him out of the nuts by suggesting games that the small squirrel cannot play. The family has to go to a restaurant for their dinner. After they pay for the dinner with twenty acorns each, the young squirrel discovers that the owner and cook is the mean squirrel and that the family have been eating their own nuts. Eventually the young squirrel confronts the mean one with his crime, and the mean squirrel repents and makes amends. The story suggests many nut activities — the games played in the story and shopping for nuts with acorns.

Ten Apples Up On Top!



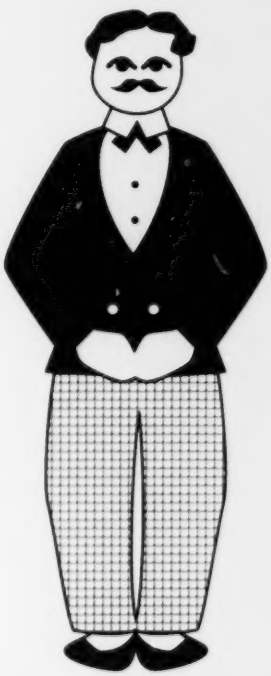


0	1	2	3	4	5	6	7	8	9	10

(The numbers can be placed on the animals to show how many apples are on top.)

Make apple strips by gluing this page to cardboard, coloring apples if desired, covering with clear adhesive plastic, then cutting. You can also duplicate this page on cardstock.

PEDDLER



MONKEY

(Redrawn from Slobodkina)

The peddler lost one cap.
Can you figure out what it looks like?

CAPS FOR SALE

by _____

Make the monkey's caps like the peddler's.

A RIDDLE

The peddler wore
6 caps.
There were
more red than
black.
There were
fewer white
than black.
There were
3 colors.
He wore 1
white cap.
Color his caps.

(Is it the same if you leave out a clue?)

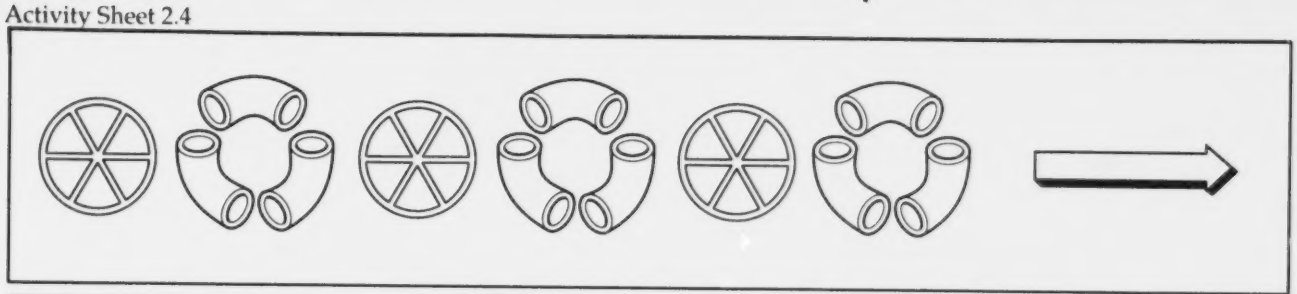
8

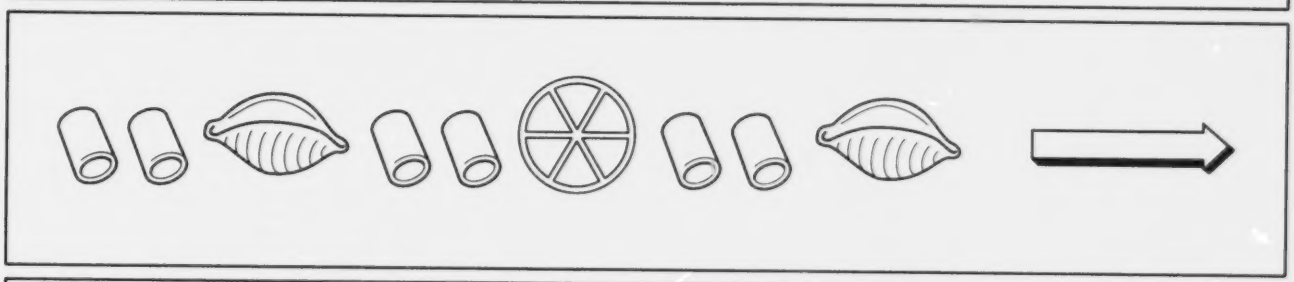
Complete the patterns.

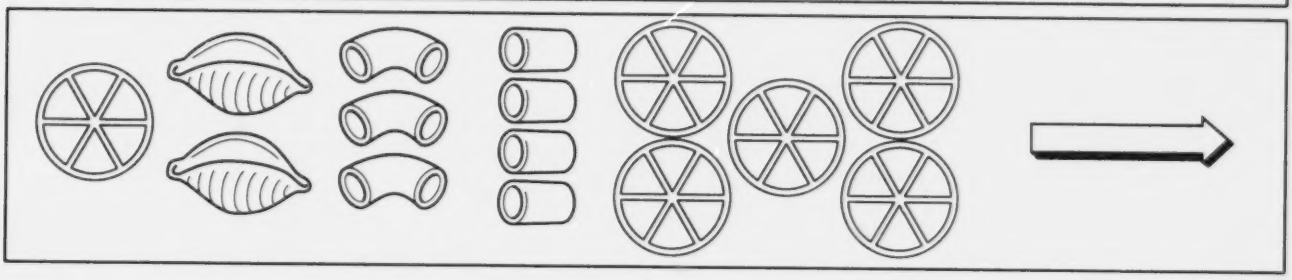
Make your own patterns.

The peddler has 5 caps.
They are white or black.
In how many ways can you color them in?
Write the sum.

Activity Sheet 2.4





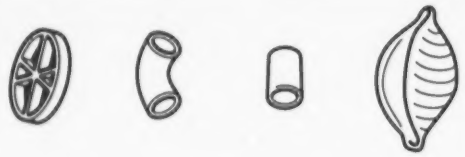


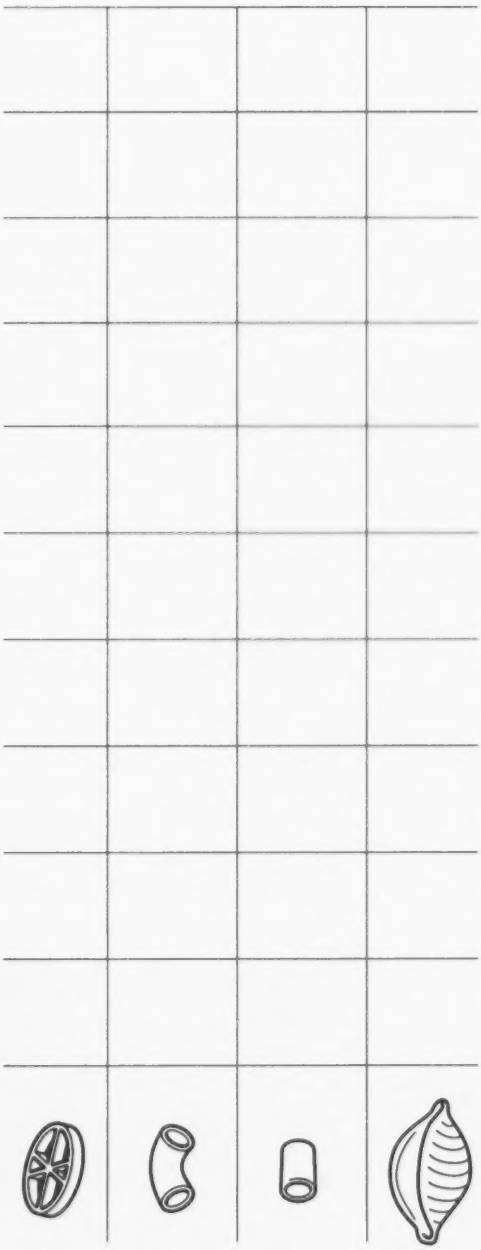
Pasta Pot Addition

Name_____

Use and .	Use and .	Use and .
Use and .	Use and .	Use and .

Name: _____

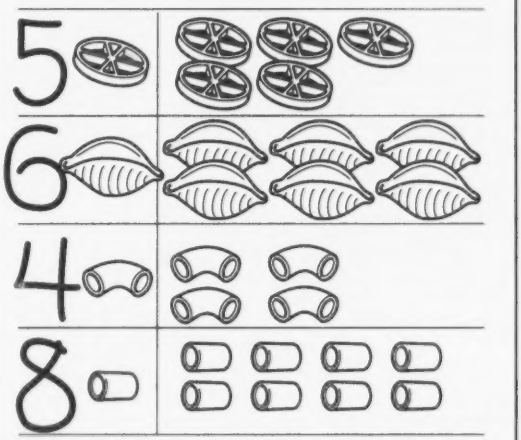




PASTA GRAPHING

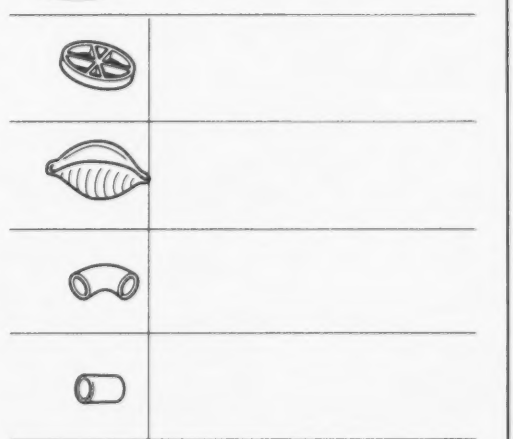
Shade in squares.

Pasta Soup Recipe



Put 1 cup water and 1 bouillon cube in a pot. Bring to a boil. Add pasta and any chopped vegetables you want. Cook until done.

Pasta Soup Recipe



Put 1 cup water and 1 bouillon cube in a pot. Bring to a boil. Add pasta and any chopped vegetables you want. Cook until done.

Pastaman

Name: _____

Take a pasta pot. Sort the pasta. Choose two kinds to put in the pots above. Pastaman always eats the pasta in the pot with more. Put him where he belongs. Write in the symbols for what you see.

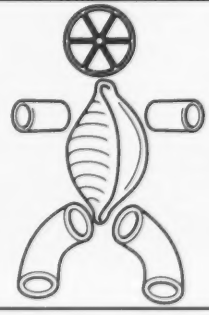
5 > 3

Use and .	Use and .	Use and .
Use and .	Use and .	Use and .

Price List

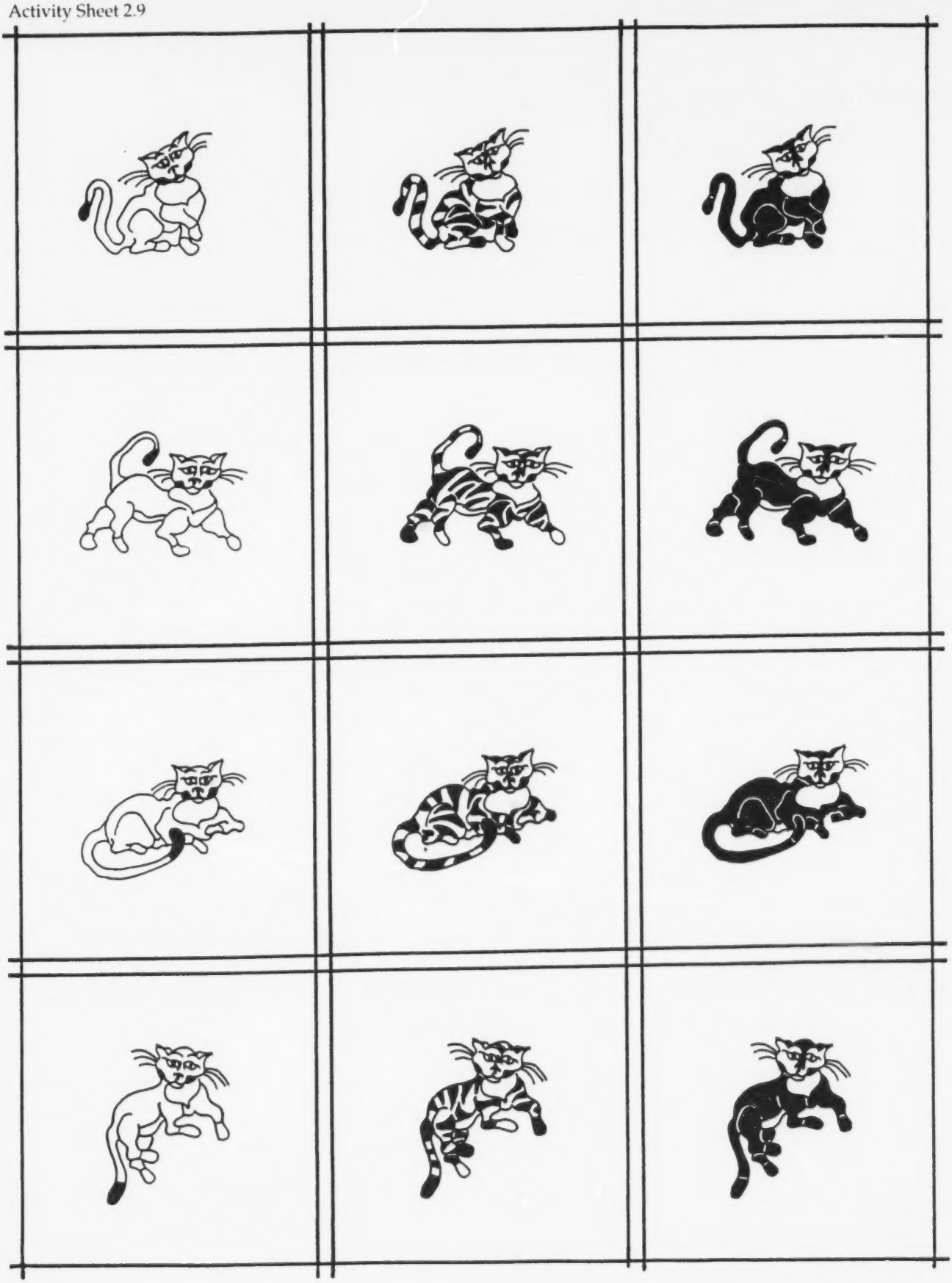
2¢ 1¢

5¢ 3¢

Buy and . How much in all?	Buy 3 pieces. How much in all?
You have 10¢. Buy and . How much change?	2 pieces cost 6¢ in all. What can they be? What else?
Could you spend 5¢ and buy 1 piece? 2 pieces? 3 pieces? 4 pieces? 5 pieces? 6 pieces?	How much would it cost to make this picture? 

Activity Sheet 2.8

Activity Sheet 2.9



3

MODEL A CREATIVE EXPERIENCE

In the *Professional Standards for Teaching Mathematics* published by NCTM in 1991, "worthwhile mathematical tasks" for students are characterized. Such tasks should be "based on...students' interests," "stimulate students to make connections," "represent mathematics as an ongoing human activity," and "promote the development of all students' dispositions to do mathematics" (p. 25). This chapter explores ways to engage children in worthwhile mathematical tasks through creative experiences suggested by a book. Many books that are identified as early "math books," including counting books and shape books, may be visually appealing, but the danger is that the mathematics is all done for the reader. However, these books can be used to motivate children to be actively and creatively involved in mathematics by creating similar books themselves or by expanding the stories or themes in creative ways.

Children's creativity can be exercised in mathematics in many ways. Geometric design, that is, the organization of space within certain constraints, invites visual creativity. Some of the appeal of poetry is related to the rhythmic patterns created at the same time as the compelling verbal images. If children are encouraged to create their own stories about number situations, they are more likely to understand the meaning of arithmetic operations on numbers. When children have an opportunity to be visually creative in designing mathematics problems and materials (as when making an attribute set such as the cat set in chapter 2), they develop a sense of ownership of the material, even if the creative process does not directly relate to mathematics. In the most general sense, children can be creative in their mathematical thinking. Throughout their study of mathematics, they should be encouraged to create and share new approaches to problems.

Most of the books suggested in this chapter involve some form of visual creativity. *Anno's Counting Book* (Anno 1977), can motivate children to create their own visual models for counting numbers. *Ed Emberley's Picture Pie* (Emberley 1984) suggests children's creation of visual models for fractions, possibly in the context of symmetry. Two shape books can encourage children to find shapes in their environment and to create new images with these shapes.

A more general way to encourage creativity when reading books with a mathematical plot is to encourage children to finish a story in their own way when it is half read. Also, children can be asked to find ways to interpret a mathematics-rich story using the mathematical tools they have available.

Anno's Counting Book

Mitsumasa Anno (1977)

Summary of the Story

This wordless book has twelve double pages, corresponding to the months of the year. The first page, labeled 0, shows a blank snowy landscape. On the next page, representing the first month, are the numeral 1 and one tree, one house, one person, ... For the second month, there are two of each item. Buildings, people, and things accumulate each month as the seasons change. Children love finding all the occurrences of each number. (Also, along the side of the page, blocks accumulate in a tower until there are ten; then on the eleventh and twelfth pages, the extra blocks start a new tower.)

Classroom Sketch (Grades K–1)

Counting
Classification

1. Show the children the book, preferably in small groups, so that each child can search for all the surprises in the pictures. As you show each page, ask what the children notice about the page and how it differs from the page before. Elicit that there are more of each object than on the page before, that the seasons are changing, that the blocks are piling up, and that there are the same number of each item (trees, houses, adults, children, trucks, etc.).

2. Ask the children to look around their own classroom. *Can you find something of which there is just one? Just two?* Continue as far as seems appropriate.
3. Show children how they might make their own pictures for each number, possibly using cutouts from magazines, precut shapes, or stamps. Children, or pairs of children, can then be given a number to work on. They can challenge each other to find all the occurrences of their number on their page.

Follow-up: A Class Calendar

Assemble twelve number pictures, from 1 to 12, into a book with a calendar for the new year, so that the picture for "1" is opposite January. Have the calendar printed or make copies. This will work best if you encourage children to draw in black marker or pencil. Children can later color in their own copies. (To give practice in writing numerals, have the children fill in the numerals on blank calendar grids for each month. Otherwise, copy pages from a printed calendar.) This could make an attractive present for children to take home to parents or other relatives at the end of the year.

Ed Emberley's Picture Pie, a Circle Drawing Book

Ed Emberley (1984)

Summary of the Book

The first part of this colorful book shows an amazing variety of pictures and patterns, all formed from a few simple elements that are fractional parts of a circle. Some designs are representational (birds, fish), some are repeating border patterns, and some are abstract, illustrating various types of symmetry. Some of the patterns are coded by colored dots to show the number of whole circles of each color that were cut up to make the patterns. The second part of the book gives step-by-step how-to instructions for making several images.

Classroom Sketch (Grades 3–5)

Fractions
Mixed numbers

1. Give each student three paper circles. Ask students to imagine that the circles are cookies or pies and discuss how one could be shared between two friends. Have students fold a circle to show one-half. Be sure students see that if the fold line does not pass through the center of the circle, the two parts are not halves (see fig. 3.1). Have students cut along the fold line (or carefully crease and tear) and label the parts "1/2". Fold for quarters with another circle, and again, make sure that students see that one folding shown in figure 3.1 does not represent fourths because the parts do not have the same area. Fold for eighths with the third circle. (If appropriate, extend to folding sixths, and challenge students to find a way to fold the circle into thirds—probably by using sixths.)

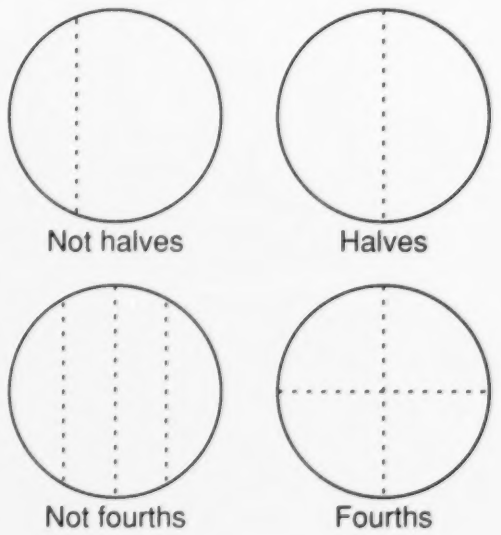


Fig. 3.1

2. Have students find ways to make the same shape using different pieces, for example, making one-half with two one-fourth pieces. As they find examples with their fraction pieces, write their findings on the board as sums or as equivalent fractions:

$$1/2 = 1/4 + 1/4$$
$$1/2 = 2/4$$
$$1/2 = 1/4 + 2/8$$

3. Ask students to arrange their pieces into an attractive pattern and share their results. Point out that if all students use all their pieces (2 + 4 + 8 = 14 in all), their patterns will all cover the same area.

4. Show Emberley's book to students and ask if there are any patterns in the book like their own. The pictures are small; so it is preferable to pass the book around or leave it available for students to look at individually. Ask them to see if they can figure out what the colored dots under each picture mean in the first few pages.
5. After all students have had a chance to examine the book, discuss the meaning of the dots. [They show how many full circles of each color were used in the pattern.] Ask students to make up similar patterns and then challenge other students to figure out how many whole circles they used. Allow students to each select a combination of colored circles, cut them into fractional parts, and make a pattern with them. (If it fits with other ongoing topics, you might ask them to make a pattern with some kind of symmetry.)

When a satisfactory pattern or picture is made, have the students glue the pieces onto a piece of construction paper. At the bottom, they can write "What circles did I use?" The answer can be placed on the back or under a flap (fig. 3.2). When the sheets are complete, allow students to exchange them.

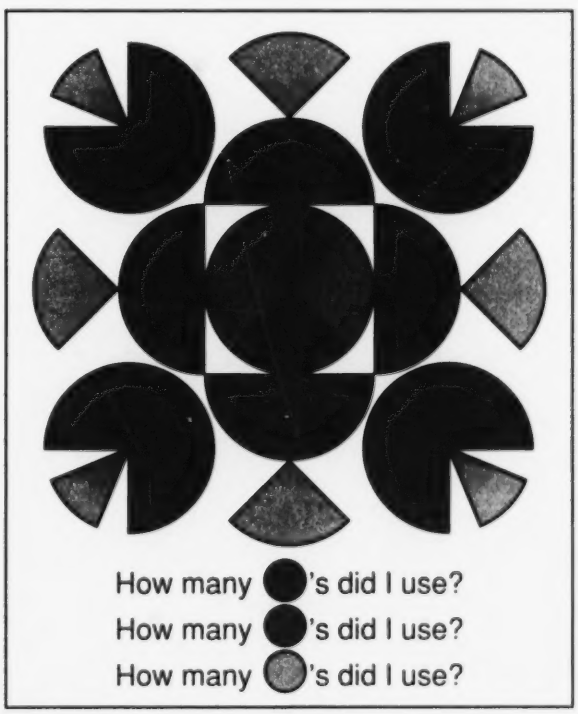


Fig. 3.2

Extension: Notation

Have the students write not only the answer to the question "How many circles did I use?" but also an equation that represents how each colored piece was cut up. (See fig. 3.3.)

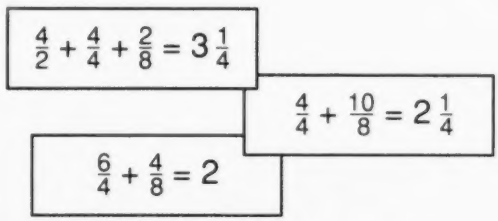


Fig. 3.3

Follow-up: Bulletin Board

These student-made patterns make an attractive bulletin board display and can be directly related to the study of mixed numbers. They also encourage students to estimate fractional parts visually. The example in figures 3.2 and 3.3 is a model. Note that for this example, you might have three equations on cards to be matched with the colors.

One, Two, Where's My Shoe?

Tomi Ungerer (1964)

Summary of the Book

In this wordless book, interesting and vivid pictures all contain shoes hidden in unexpected places.

My Very First Book of Shapes

Eric Carle (1974)

Summary of the Book

This is a wordless, split, board book, where the lower pages have a colorful picture and the upper ones have corresponding shadows but in a different order. Children are to flip the pages until they match the picture to the shadow.

Note: These books have been chosen from the many available shape books because they do not fit into the pattern that encourages children and teachers to think that "learning your shapes" means naming just circle, square, triangle, and rectangle and learning these imperfectly! Children learn the names of shapes that

are useful to them. Early experiences with shapes should allow children to explore the rich variety of shapes around us. Either of these books, or other similar ones, could be used to initiate the following activities:

Classroom Sketch A (Grades K–2)

Matching shapes

1. Show the chosen book to the children This should be done in a small enough group so that each child can find the hidden shapes.
2. Draw some outlines on paper and encourage the children to look around the classroom for that shape. Try some varied shapes and let the children try also. Discuss how it is sometimes difficult to draw the shape exactly.
3. Introduce another way to ask similar questions. Show a picture with many embedded shapes and place a sheet of acetate on top of it. Have a child identify a shape in the picture and trace over it with a pen designed for overhead transparencies or with a china marker. Then remove the sheet and ask other students to see if they can find the shape in the picture. They can check by putting the acetate back on top. Make more puzzles like this for the children to use independently, possibly in a learning center.

Follow-up: Learning-Center Activity

Have several find-the-shape puzzles available. Children can make their own, but you may need to help them make the lines exact. Puzzles can also be made by cutting the shapes out of colored transparent acetate—for example, a colored report folder. An example using a quilt pattern is on activity sheet 3.1.

Classroom Sketch B (Grades 1–3)

Shapes in the environment

Repeat steps 1 and 2 from above.

3. Make several copies of a certain shape (perhaps a rectangle) on paper or the chalkboard. Ask children what this shape could be in a picture. Encourage ideas, and for each idea, add to the shape drawing to make the item suggested (perhaps a house, a truck, a box).
4. Explain to the children that they will make similar pictures. Show a sample worksheet (fig. 3.4). Children are to make a picture out of the shape on the lower half of the sheet, then fold the page along the line. Another child can look at the picture, try to find the indicated shape, and then hold the folded paper up to the light to see where the shape is hiding.

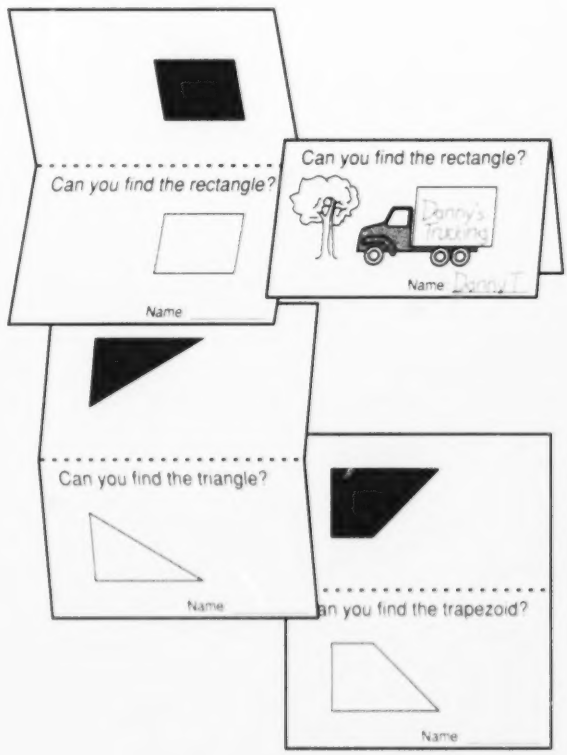


Fig. 3.4

Follow-up: Worksheet

Make several sheets like those in figure 3.4. As children learn geometry vocabulary, you can expand the collection of shapes involved. It is easy to make the pages so that the shadow lines up with the outline, when folded. Fold a blank piece of paper in half, then insert a folded sheet of carbon paper with the carbon side out. Draw the shape you want, then open the page and shade in one copy of the shape. Reverse the fold to use the activity sheet.

Other Books

Visual Creativity

Many traditional counting books can serve as models for young children to design their own theme-

related books. They might want to draw pictures for their own counting books related to a current topic of interest (dinosaurs, pets, food). The books could also be illustrated by pictures cut from magazines. Similarly, shape books lead naturally to children's looking in their environment and in pictures for categories of shapes to organize into their own books.

The books about quilts suggested in chapter 1 for establishing a context for a mathematical activity could also be used in ways that would fit this chapter. Give children multiple copies of a few simple shapes and encourage them to use the shapes to fill a square in some way. Then see what overall effect they can create by repeating that square many times.

Creative Writing and Drama

Stories with a strong mathematical component to the plot, such as ***A Grain of Rice*** (Pittman 1986) (see chapter 4), can give children an opportunity for mathematical creativity if the reading of the story is stopped part-way through and the children are asked to complete the story. Children might also dramatize such a story. ***How Big Is a Foot?*** (Myllar 1962), described in chapter 6, would make an entertaining performance because several times a question is repeated through a chain of command (which could consist of more than the four characters in the story).

Creative Interpretations

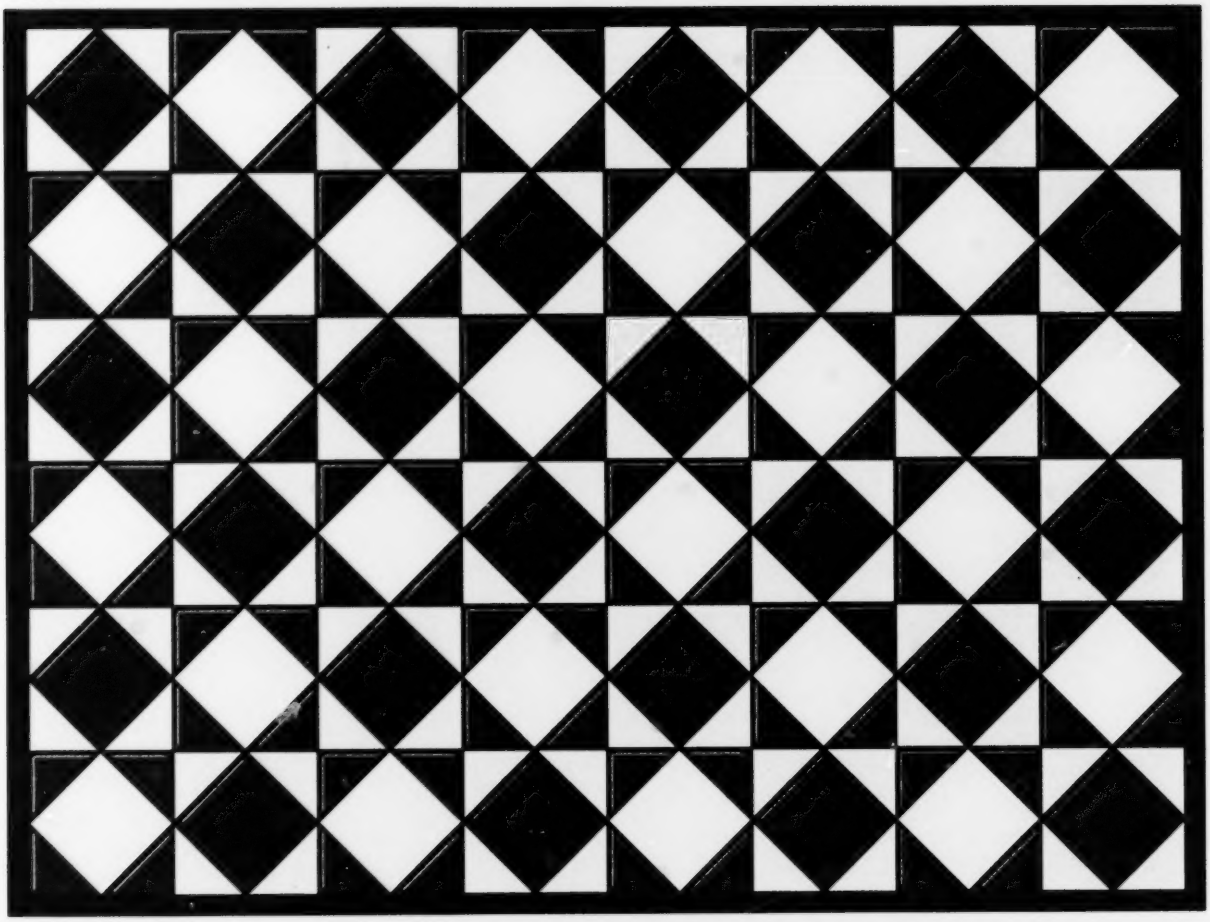
Some stories lend themselves to the following challenge: Find different ways to interpret the story mathematically.

The Twelve Days of Christmas. This book by Jack Kent (1971)is mentioned in chapter 4 as a book that provides a context for posing some interesting problems. But another way to use it that encourages mathematical creativity is to divide a class into groups and assign each group a mathematics-related way to interpret the song. In a fifth-grade class where this was done, one group thought of all the different ways graphs could be made about the song. A second group designed mathematics-related games based on the song. A third group designed tree decorations, one for each day, with the same number of sides as the day (for example, for the fourth day they made a tetrahedron and drew a calling bird on each face). A fourth group, consisting of children who knew something about musical notation, examined the music and analyzed the patterns of the notes—duration and pitch. The results of this somewhat open-ended project demonstrated a lot of creative thinking by the students!

BEST COPY AVAILABLE

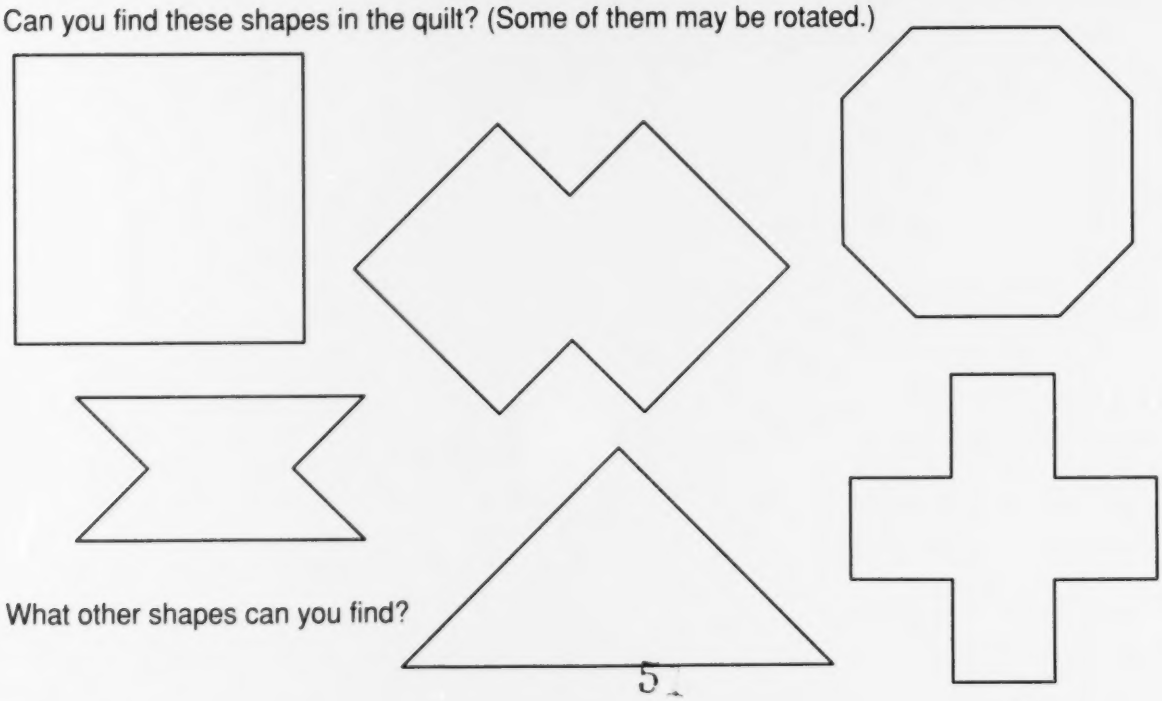
Activity Sheet 3.1

A Quilt



This is a patchwork quilt.

Can you find these shapes in the quilt? (Some of them may be rotated.)



What other shapes can you find?

4

POSE AN INTERESTING PROBLEM

The Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) presents "becoming a mathematical problem solver" as a fundamental goal for students (p. 6). Much of that document illustrates how problem solving as a process can be incorporated throughout children's study of mathematics. Most of the classroom sketches in this booklet have some aspect that one could call "problem solving." This chapter gives examples of books that in themselves pose or suggest a problem worthy of extended investigation.

Some books involve a mathematical problem as an integral part of the plot. A particularly good example is *A Grain of Rice* (Pittman 1986), in which a classical problem about growth and large numbers is embedded in a story about a young man in love with a princess in ancient China. This story can be read on many mathematical levels because although the conditions of the problem are clearly stated, little is said about the ways to think about and compute the large numbers in the story. In fact, there are many strategies that can be used to formulate and approach the problem.

Sometimes a book sets up a situation where there are natural mathematical questions to be asked, even if the story doesn't ask them. An example is Jack Kent's interpretation of the song "The Twelve Days of Christmas." The book leads directly to a situation—the presentation of a vast array of gifts—that can lead to questions at a wide range of levels.

In some books, an explicitly stated problem or situation is presented, but only as a small part of the plot. For example, the Roald Dahl books, *The Witches* (1983) and *Matilda* (1988), give data about the heartbeat rates of a mouse and other animals. A few questions from the teacher or students can expand such segments of a book into a problem for extended exploration.

There are many types of books that pose or suggest problems. The books of Mitsumasa Anno often pose mathematical problems directly. Historical books suggest problems about time, sports books suggest problems about measurements and rates, and books about change in scale suggest problems of ratio. Once you have shown students a few examples of how mathematical problems can arise in literature, you might leave it as an open challenge for them to find more such examples in their own reading. You may be surprised by the variety of ideas that arise as the students begin to look at both literature and mathematics in a new light.

The Twelve Days of Christmas

Jack Kent (1971)

Summary of the Story

> "On the first day of Christmas, my true love gave to me, a partridge in a pear tree.
>
> On the second day of Christmas, my true love gave to me, two turtle doves and a partridge in a pear tree...."

And so it continues with three French hens, four calling birds, five golden rings.... In this amusingly illustrated version of the song, a boy brings his girlfriend all the items in the song, starting at the beginning each day. So, for example, by the end of the third day she has three partridges in pear trees, four turtle doves, and three French hens. Understandably, by the last day, the girlfriend has fled!

Classroom sketch (Grades 2–6)

Counting
Number patterns

1. Read the story to the children. If desired, children can suggest their own items for the gifts. (Teachers might want to say, "On the first day of the holidays...," instead of, "On the first day of Christmas....") As the days progress in the story, ask if the girl seems happy with the gifts. Why or why not? Ask what questions could be asked about the picture of, say, the seventh day.
2. Pose a question about a page according to the students' level. Many questions are suggested by the song. Select appropriate questions for your

students, and allow them to work out their answers in small groups.

a) *How many gifts are given on the seventh day? On the tenth? On the twelfth? Is there a pattern? Can you find a way to predict how many gifts would be given on, say, the fiftieth day if the gift-giver persisted?*

If students have investigated triangular numbers, they might recognize this sequence, 1, 3, 6, 10, 15,..., and realize that the pattern of differences between successive triangular numbers is given by the counting numbers (see fig. 4.1).

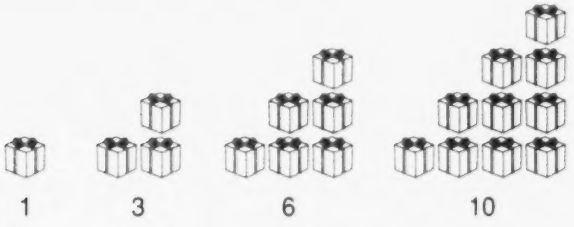


Fig. 4.1

Older students might figure out that to get the seventh triangular number, they can multiply 7 by 8 and divide the result by 2, and similarly to get the nth triangular number, you can multiply n by $n + 1$ and divide the product by 2. A visual explanation of this procedure is shown in figure 4.2.

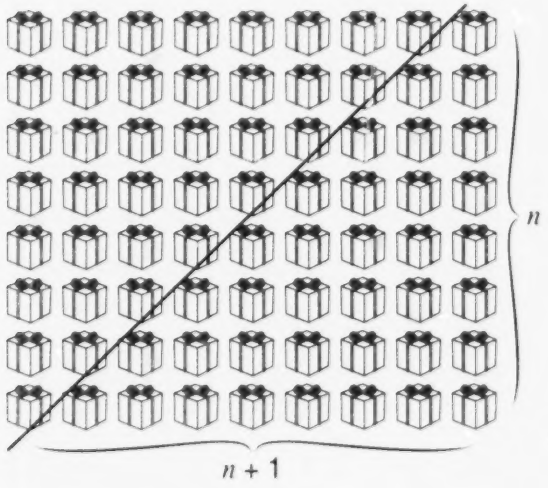


Fig. 4.2

b) *How many of each item have been given by the twelfth day?* [12 × 1 partridges, 11 × 2 turtle doves, 10 × 3 French hens, etc.] *Of which item are the most given?* [7 × 6 = 42 of the item introduced on the sixth day are given, the same number as 6 × 7 of the item introduced on the seventh day.]

c) *How many items in all have been received by the twelfth day? By what day had the girl received 100 presents?*

Note: Chapter 3 describes some other uses of this book for creative mathematics experiences for children.

The Witches

Roald Dahl (1983)
Illustrated by Quentin Blake

Matilda

Roald Dahl (1988)
Illustrated by Quentin Blake

Summary of the Stories

In the first of these books, a young boy accidentally wanders into a witch convention and hears of a horrifying plot. He is captured and turned into a mouse, but in spite of this, together with his grandmother, he takes on and triumphs over the witches.

In the second book, a sweet and sensitive young girl, born into a crooked and unpleasant family, is in fact a genius with extraordinary powers. She manages to triumph over her family and the horrible headmistress of her school with the help of the one person who understands her—her sweet but oppressed teacher, Miss Honey.

These books are presented here together because both mention some interesting data. In *The Witches*, in the chapter entitled "The Heart of a Mouse," the grandmother tells the young hero, now a mouse:, some almost unbelievable information about himself:

> "The heart of a mouse," she said, "and that means your heart, is beating at the rate of five hundred times a minute! Isn't that amazing!"
>
> "That's not possible," I said, opening my eyes wide.
>
> "It's as true as I'm sitting here," she said. "It's a sort of miracle."
>
> "That's nearly nine beats every second!" I cried, working it out in my head.
>
> "Correct," she said. "Your heart is going so fast it's impossible to hear the separate beats. All one hears is a soft humming sound." (Pp. 189–90)

(Reprinted by permission of Farrar, Straus & Giroux, Inc.)

The following excerpt is from *Matilda*.

> Matilda, who was perched on a tall stool at the kitchen table, ate her bread and jam slowly. She did so love these afternoons with Miss Honey. She felt completely comfortable in her presence, and the two of them talked to each other more or less as equals.
>
> "Did you know," Matilda said suddenly, "that the heart of a mouse beats at the rate of *six hundred and fifty times a second?*"
>
> "I did not," Miss Honey said smiling. "How absolutely fascinating. Where did you read that?"
>
> "In a book from the library," Matilda said. "And that means it goes so fast you can't even hear the separate beats. It must sound just like a buzz."
>
> "It must," Miss Honey said.
>
> "And how fast do you think a hedgehog's heart beats?" Matilda asked.
>
> "Tell me," Miss Honey said, smiling again.
>
> "It's not as fast as a mouse," Matilda said. "It's three hundred times a minute. But even so, you wouldn't have thought it went as fast as that in a creature that moves so slowly, would you, Miss Honey?"
>
> "I certainly wouldn't," Miss Honey said. "Tell me one more."
>
> "A horse," Matilda said. "That's really slow. It's only forty times a minute."
>
> This child, Miss Honey told herself, seems to be interested in everything. When one is with her it is impossible to be bored. I love it. (Pp. 230–31)

(Used by permission of Viking Penguin, a division of Penguin Books USA Inc.)

Classroom Sketch (Grades 4–6)

Rate
Forming and verifying hypotheses
Graphing

Note: This activity would be most appropriate after most children have read both of these books (or have seen the movie of *The Witches*) so that the plots can be summarized and only the segments above read.

1. Read the two excerpts. Ask students to make notes about the information presented in the two books.
2. Ask students if they notice any inconsistencies. *Which story do you think is correct? Why might an error have occurred?* [It seems likely that in Matilda the rate should have been "six hundred and fifty times a minute," since the later rates are in minutes. This is not impossibly far off from the data in *The Witches*.] *Might both figures for the mouse heartbeats per minute, 500 and 650, be correct? Would human heartbeats vary this much? What affects heartbeat rate?* (Age? Size? Exertion? Emotion?)
3. Experiment by measuring heartbeat rates per minute in the class. *Is there much variation in the class? How can an individual change his or her heartbeat?* Students might propose various

experiments, for example, measuring the rate before and after running in place for three minutes. Students should collect data and make a graph of their results, possibly as a double bar graph or by percentage increase in rate. Discuss whether the discrepancy in the Dahl books (assuming the correction in *Matilda*) would fit in the range found in the classroom experiments.

4. Return to the data about other animals. On the basis of the information about the mouse, the hedgehog, the horse, and the human, can students form a hypothesis about how the heartbeat rates of animals relate to some other attribute? [They may suggest that smaller animals have faster heartbeats and larger animals have slower heartbeats.] Discuss how their hypotheses could be verified.

Follow-up: Collecting Data

Have children collect data related to their hypotheses about heartbeat rate. Can they measure the heartbeat rates of their pets directly?

Follow-up: Relating Heartbeats to Longevity

In the chapter of *The Witches* mentioned above, the grandmother discusses with the mouse-boy how long an ordinary mouse will live (about three years) and how long a mouse-person would be expected to live (at least three times as long). They decide that the grandmother and the mouse-boy will continue to live about the same amount of time. This information can lead to further questions and investigations. *For example, whose heart will have beaten the most if the mouse-boy and the grandmother live to the same year? Might it be true that the rate of heartbeat is related to the life span of the animal?* Students might investigate how many times a human's heart can be expected to beat in

a lifetime and how many times a pet's heart will beat. A useful source of data is the *Guinness Book of Pet Records* by Gerald L. Wood (1984), in which these average life spans for pets appear:

Dogs (smaller dogs longer than large ones)	8–15 years
Cats—male	13–15 years
—female	15–17 years
(strays 6–8 years)	
Rabbits (large ones up to 15 years; wild ones 18 months)	8 years
Rats (wild ones 1-2 years)	3 years
Mice	2–3 years
Goldfish (fancy ones 14 years)	25 years
Guinea pigs	4–5 years
Gerbils	3–4 years

Note: Roald Dahl seems to have liked mathematics because almost all his books have segments that relate in some way to mathematics. In *Matilda*, the chapter entitled "Arithmetic" describes how Matilda demonstrated her extraordinary powers of mental arithmetic but was accused by her father of cheating. The chapter entitled "Miss Honey" describes Matilda's first day in school and how her teacher discovers her arithmetic powers. Both of these segments could be used to discuss what is meant by mental arithmetic and what sorts of calculations an ordinary person might be expected to do in her head.

A Grain of Rice

Helena Clare Pittman (1986)

Summary of the Story

A young man, the son of a farmer, comes to the court of the emperor of China and asks for the hand of the emperor's daughter, the princess, in marriage. This is considered impossible, yet the young man impresses the court with his boldness and quickness and so is allowed to work in the palace. The princess falls in love with the young man, but the emperor wants her to make a more suitable match. The princess falls ill, and the emperor promises any reward to whoever can cure her. The young man provides the cure and again asks for the princess's hand in marriage. When he is again refused, he asks instead for a grain of rice. When the emperor proclaims that this is too small a reward, the young man says, "If his majesty insists, he may double the amount of rice every day for a hundred days." It soon becomes evident that this is an enormous amount of rice—by the second month there would be no more rice left in China. The emperor decides to make the young man a prince and let him marry the princess.

Classroom sketch (Grades 4–6)

Number patterns
Using calculators
Estimating large numbers
Exponents

1. Read the story to the students. If desired, stop at the description of what the young man receives on the seventh day, sixty-four grains of rice, and let children write their own endings to the story.
2. Discuss the students' story endings. Read the actual ending. Raise the question of how many grains of rice would be given in 100 days. Have students figure out how many grains of rice the young man receives on each day. These numbers can be expressed as follows:

   1, 2, 4, 8, 16, ...

   Or they can be expressed in terms of how they are calculated:

   $1, \quad 2, \quad 2 \times 2, \quad 2 \times 2 \times 2, \quad 2 \times 2 \times 2 \times 2, \quad \ldots$

   If appropriate, the exponential notation can be used:

   $1, \quad 2^1, \quad 2^2, \quad 2^3, \quad 2^4, \quad \ldots$

   In fact, this is a nice place to point out that, following the pattern, "2^0" is a good name for "1".) Students might use calculators to find out how many grains of rice the young man has received. For how many days can they do the calculation on their calculators? [Only 27 on an eight-digit calculator because the numbers become too large]. Note that an easy way to do this calculation on most calculators is to press $2 \times = = =$ .... After the third day, when you push as many keys in this sequence as the number of the day, the result shown on the calculator is the number of grains of rice received on that day! For example, if you perform the key sequence $2 \times = = =$, you have pressed a key five times and the calculator shows 16, which is 2^4, the number of grains of rice received on the fifth day.)
3. Ask how much rice the young man had accumulated, say, by the twentieth day. Even using

a calculator, this calculation is tedious unless students notice a pattern. Encourage them to start from the first day and record the sums:

$$\begin{aligned}
1 &= 1\\
1 + 2 &= 3\\
1 + 2 + 4 &= 7\\
1 + 2 + 4 + 8 &= 15\\
1 + 2 + 4 + 8 + 16 &= 31\\
1 + 2 + 4 + 8 + 16 + 32 &= 63\\
1 + 2 + 4 + 8 + 16 + 32 + 64 &= 127
\end{aligned}$$

By this point, the students may well notice the proximity of 63 and 64, 31 and 32, and 15 and 16, and they may note that on the seventh day, the total number of grains of rice, 127, is one less than $128 = 2^7$, and in general on the nth day, the total is $2^n - 1$. Using this information, they should be able to answer the original question quickly with their calculators.

4. A natural question now is how much rice these numbers represent. This lesson might end with an estimation contest, in which students are shown a small clear container (such as a pill bottle) full of rice or beans and are asked to individually make an estimate of the number of grains of rice that it contains. Ask students to examine the container, doing anything they want short of opening it, and put their written estimates in a box.

Follow-up: Analyzing Class Estimation

Discuss how to determine how well the class estimated the amount of rice in the pill bottle. *Did anyone have exactly the right number?* How can students determine easily what fraction of the class overestimated? Have a group of students arrange the estimates in order. *What was the greatest estimate? The least? What was the middle value?* Analysis of a class's estimates in this way provides a natural way to introduce or review statistical concepts, such as range, and types of average, such as median, mode, and mean.

Follow-up: Estimating and Counting Very Large Numbers

Discuss the strategies that students used to estimate the number of grains of rice in the container. Some may have counted the grains that could be seen on the bottom, estimated how many layers there were, and multiplied.

Discuss how the rice can be counted to verify the estimates. Some methods will lead to better estimates. Try the following:

- Use area: Spread the rice out on 2-cm grid paper in a single layer, adjusting it to make a rectangle. Count the grains in one square and multiply.
- Use capacity: Fill a small container such as a cap from a small bottle. Count how many grains fit in it and then how many capfuls fill the container.
- Use weight: Count out 100 grains and weigh them (if you have a fine balance). Weigh the total.
- Use fractional parts: Divide the contents of the container in half, then in half again, and again,..., until the pile is small enough to count easily.

Other methods will lead to an "exact" count (although a few grains are sure to be dropped or missed). The most efficient way to count the grains is to divide them up approximately evenly among groups of students, and have each group count its grains. To reinforce place value, give each group a sheet of squared paper (again, 2-cm squares would work well) and have them place ten grains in each square. Then add the results of each group.

Show a bag of rice (five or ten pounds) to the class. Ask them to estimate how long it would take for the bag to be used up in the story. Encourage them to use what they found about the number of grains in the container that they counted.

Follow-up: Estimating Large Numbers and Exploring Exponents

Ask how many grains of rice would have been accumulated by the one-hundredth day. Students should realize that there will be $2^{100} - 1$ grains of rice. To get an impression of how much this is, some sort of estimation is appropriate. When calculating the powers of 2, students may have noticed that $2^{10} = 1024$, which is very close to 1000. Thus they can figure out that $2^{100} = 2^{10} \times 2^{10} \times 2^{10} \times \ldots \times 2^{10}$ (ten times), which is about 1000^{10}. If they try to write this out as a product of 10s, they will see that it equals $10 \times 10 \times 10 \times \ldots \times 10$ thirty times. This is an enormous number.

Follow-up: Further Investigations

Pose the question of how much the emperor's gift would cost up to the twentieth day. Students might do some research into the price of different types of rice in different-sized containers in local shops. What is

the least expensive way that an emperor living in their neighborhood could buy this gift?

Note: If you have accumulated rice in the course of this project, you might want to do some further investigations! The package contains a lot of other quantitative information about which you or your students could raise questions. They might look at the recipes, the price, the number and cost of servings in a container. If you cook some rice, you might notice changes in volume and weight of cooked versus uncooked rice. Students might try a taste test on various types of rice. Finally, examine how the nutritional content varies from one type of rice to another.

Other Books

Mathematics

Many excellent books for children raise mathematical problems. Marilyn Burns has written several, including ***The I Hate Mathematics! Book*** (1975) and ***Math for Smarty Pants*** (1982). However, the focus of this booklet is on those books where mathematics is embedded in what could be called children's literature. Mitsumasa Anno has written and illustrated many books with significant mathematical content in the story. Two whose use fits in this chapter are the following:

Anno's Hat Tricks (Nozaki 1985)

Socrates and the Three Little Pigs (Mori 1986)

These two books illustrated by Mitsumasa Anno present logical reasoning puzzles and combinations and permutations in a manner appealing to children in grades 2 to 6. The books need to be read with frequent pauses so that children can think through the problems posed.

Many classic magic tricks depend on mathematics. Some of these are presented in a very appealing way by the magician Blackstone in the television program "Square One TV," produced by the Children's Television Workshop.

Math-a-magic, Number Tricks for Magicians, by Laurence B. White and Ray Broekel (1990) and illustrated by Meyer Seltzer, has many more examples of magic tricks and mathematics.

History

Historical novels suggest problems concerning the passage of time and how quantitative aspects of our lives change over time. Such books suggest the construction of time lines to show events in the book and in earlier or later times than the book.

And Then What Happened, Paul Revere? (Grades 3–6) This book by Jean Fritz (1973) about Paul Revere contains many interesting numerical facts. For example, it states that Paul Revere rode on horseback from Boston to Philadelphia and back at the rate of 63 miles a day and that he arrived home on the eleventh day. Students might check these figures with distances in a current atlas (323 miles between Boston and Philadelphia). This excerpt might also suggest looking at the rate of travel by different means throughout history. How long does it take to travel this distance now? (Students could look at airline, train, and bus timetables and the driving times projected on maps.) They might try to find out how long the trip would have taken 50 years ago, or 100 years ago. Many related graphs can be constructed.

Sports

Books about sports appeal to many students and often contain a wealth of numerical data. A biography of a famous sports figure suggests problems on gathering and interpreting statistics. A good example might be Jesse Owens because students can time themselves as they run fixed lengths and figure out how much more quickly Owens would have covered a certain distance than they can. It is also interesting to see how the Olympic records for running have changed over time and to anticipate what will happen to these records in the future.

Jesse Owens, by Tony Gentry (1990), is one possible biography.

Change in Scale

What would your life be like if you were very much smaller or very much larger? This question is raised in many classic works of children's literature: ***Stuart Little*** by E. B. White (1945), ***The Borrowers*** and its sequels by Mary Norton (1953), ***Alice in Wonderland*** by Lewis Carroll (1985), and ***Gulliver's Travels*** by Jonathan Swift (1947). Also, of course, the idea of change of scale is a staple of science-fiction movies.

Any of these works can promote the investigation of ratio and scale. After reading a book about a giant, you can show a bracelet and ask, "If this were a ring for the giant, how tall would she be?" Or, if you have been reading about a tiny person, show a piece of dollhouse furniture and ask, "If this were a chair for this small person, how tall would he be?" You could also ask students to design a table and chair for someone who is 8 cm tall. This investigation might encourage critical viewing of science-fiction movies, such as ***Honey, I Shrunk the Kids*** from Walt Disney Pictures (1990), to see if the scale changes seem to be consistent. Curriculum units on change of scale have been built around *Gulliver's Travels* by the Wiskobas Project in Holland (described in *Three Dimensions* by Adrian Treffers [1987] and by the Journeys in Mathematics Project, currently under development at Education Development Center in Newton, Mass.). Some very interesting questions can be raised about what happens to the volume or weight of an organism if it is uniformly shrunk or expanded. These considerations have numerous implications in biology, as outlined in "On Being the Right Size" by J. B. S. Haldane in *The World of Mathematics*, Vol. 2 (1956).

5

PREPARE FOR A CONCEPT OR SKILL

Many of the uses of children's literature described in previous chapters are open-ended. A book may suggest a variety of mathematical activities, which may lead students to mathematical topics far from the original content of the book. Chapters 5, 6, and 7 concern books (or parts of books) that are more focused in terms of mathematics—they can fit neatly into the teaching of specific skills or concepts. The standards in *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) rest on the assumption that curriculum should be conceptually oriented. The examples described here illustrate how literature can provide a context for concept development. They have been separated into three chapters according to how they fit into the stages of a child's learning about the mathematics concept.

As children learn about mathematical topics, three distinct types of activities are appropriate. First, before the topic is formally introduced, children should be exposed to it in terms of their previous experience. As stated in the *Curriculum and Evaluation Standards*, "a conceptual approach enables children to acquire clear and stable concepts by constructing meanings in the context of physical situations and allows mathematical abstractions to emerge from empirical evidence" (p. 17). Sometimes this abstraction can happen considerably before the topic appears in the standard curriculum. For example, the book *The Doorbell Rang* (Hutchins 1987), which is a story about the division involved in sharing a plate of twelve cookies, is well loved by first graders, who will probably not encounter the formal terminology and symbolism for division until late second or third grade. Preparatory experiences are usually characterized by active exploration, whether with manipulative materials or more abstract tools, building on the language and experiences that children already have and laying the groundwork for new abstractions to follow. Books that suggest these sorts of experiences are in this chapter, chapter 5.

At some point this preparatory experience should be analyzed explicitly and associated with a new language and symbolism. At this stage, while the new concept or skill is being developed and explained, reference must still be made to the learner's base in experience. Books that offer a context for establishing this connection are described in chapter 6.

A child who has developed a sound conceptual base then needs to apply or practice mathematical understanding or skills, perhaps without reference now to the original experiential base. Mathematical skills at all levels should be practiced in the context of interesting mathematical problems or real-world contexts. Books that provide suitable contexts to practice or review a skill or concept are described in chapter 7.

A given book might be used at more than one of these stages. For example, *The Doorbell Rang* is sometimes used by third-grade teachers when they describe the sharing with the formal language and symbolism for division. Thus, books mentioned in these three chapters should be considered in the light of possible adaptation to other stages.

Many books intended for preschool or primary grades, described as "concept books," are intended to lay the groundwork for such mathematical concepts or processes as classification, sequencing, comparison of size, and patterning and number, and thus these books fit in chapter 5. The challenge for the teacher using these books is to make sure that the children are as actively involved as possible, with a "minds on," even if not a "hands on," involvement.

Not all books suitable for preparing for a mathematical concept belong in the primary grades. An example is given of how *Charlie and the Great Glass Elevator* by Roald Dahl (1972) can give children an exploratory experience with negative numbers before they consider them formally.

The Doorbell Rang

Pat Hutchins (1987)

Summary of the Story

Twelve cookies are to be shared by two children, but the doorbell keeps ringing; so what at first

must be shared between two must be shared among four, six, twelve.... Luckily, Grandma comes with more cookies!

Classroom Sketch (Grades 1–2)

Preparing for division

1. Read the story to the children. Discuss situations in which children have to share things.
2. Ask the children to act out the story. Use magnetic chips to represent cookies. Start with two children and twelve cookies and have children act out the sharing of the cookies. If each child has a small pizza pan, the children can hold up their "cookies" so that all can see what they have. Each time the doorbell rings, have more children join in and share the cookies anew.
3. Act out the story again, but this time do not follow the story line exactly. Ask what would happen if different numbers of children arrived. Allow children to discover that if there are two, three, or four children to share the cookies, there are no leftovers, but if five children try to share the twelve cookies, each will get two and there will be two cookies left over. What should be done with those cookies? [Children might propose that they be saved for the grandmother or that they be broken into pieces. A child might suggest dividing each into five parts and giving each child two of these parts.]
4. Let the children act out other stories, working in small groups and using counters to represent the cookies. Give each group a bag of "cookies" and ask them to share them among two, three, four, ..., finding out when they can share them exactly and when there will be leftovers.

Note: When doing this activity with young children, keep the language about division informal. The objective is to have children practice counting and experience the activity of sharing, making sets of the same number.

If you are using this activity when the symbol for division is introduced, model the language and symbols—when twelve cookies are shared among four people and each gets three cookies, we say "twelve divided by four equals three," or "12 ÷ 4 = 3." This use of the book would fit in the next chapter.

Reflections

Ann Jonas (1987)

Summary of the Story

This book describes a child's day trip to the beach. What is unusual is that the book is read first in one direction, then is turned upside down and read again from the back to the front. In this process, some images are seen as one thing the first time through but are interpreted in an entirely different way on the return trip, when seen upside down. For example, a motorboat in the water becomes a plane in the sky, and sailboats in the water become kites in the sky. However, other images—those with a horizontal line of symmetry—look exactly the same from either direction. Some cars and buildings are drawn this way, and two pages show a shoreline perfectly mirrored in the water.

Another book by the same author, ***Round Trip*** (1983), has the same idea but is entirely in bold black-and-white images, whereas *Reflections* is in color. Some pages of *Round Trip* have letters or numerals that can also be read as letters or numerals upside down.

Classroom Sketches (Grades 2–4)

Line symmetry
Rotational symmetry

1. Read the book to the students. Pose the following questions: *How can a picture still be a recognizable picture when turned around and seen upside down? Does this happen with many pictures? When do the objects look just the same from either direction?* Try to find some examples in the book. *When do the objects become different?* Find some examples. *Why is the book called* Reflections? *On the cover, is the way the title is written really a reflection?* [Actually not—the word is rotated.] Check with a mirror.
2. Have children classify a page of pictures, letters, or numerals (such as those on activity sheet 5.1). Find all the items that look exactly the same upside down. Have the children place a mirror on each picture to see if the picture changes. *Can you find some places to put a mirror so that you see the same picture?* Have children add to the collection of pictures, by cutting images from magazines or by drawing their own.

[Letters in the left-hand loop have at least one *line of symmetry*, which means that if you put a mirror on the line of symmetry, you will see the original letter. It also means that if the letters are written on a transparent material, you will see the same letter if you flip it over the line of symmetry. Letters in the right-hand loop have *rotational symmetry*, with the point of rotation being in the shapes, which means that if you rotate them (in this case by 180 degrees), they will look just the same. Letters outside both loops have neither type of symmetry.]

Do not expect children to use the formal language for symmetry at first. Encourage them to find their own words to describe the characteristics of the letters.

Follow-up: Sorting Shapes for Symmetry

Draw large letters on cards. Play a guess-my-rule game with the cards as shown in figure 5.1. *Why are the letters arranged this way in the loops?* Have students try to verbalize a description of each region. [See page 22 for a similar game where pieces are placed in overlapping loops in "Classification in Venn diagrams."]

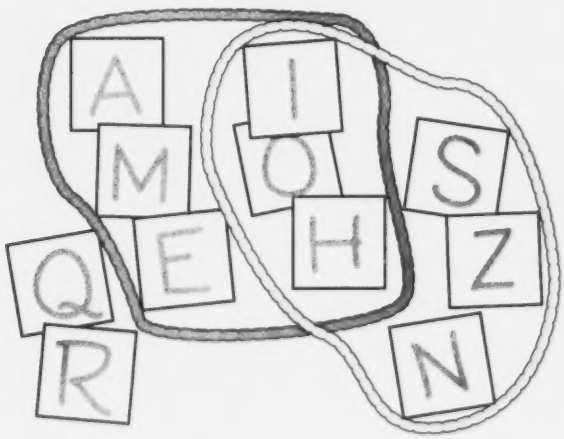


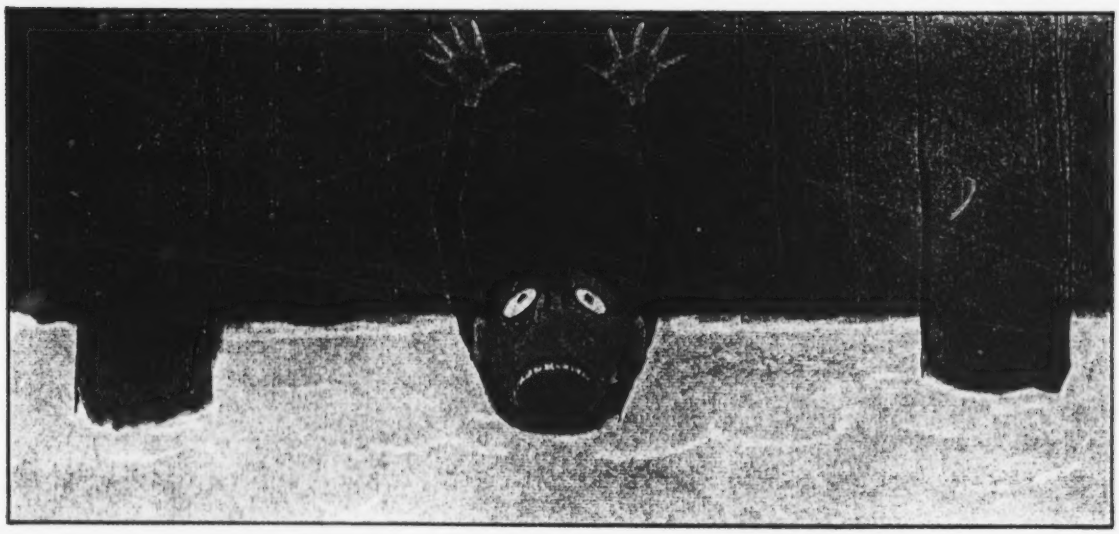
Fig. 5.1

Note: Other books that can be used to introduce symmetry are ***Magic Mirror Book*** (1970) and ***Magic Mirror Tricks*** (1971) by Marion Walter (also available in earlier printings by different titles). In these books, children experiment with placing the plastic mirror included with the book on pictures to alter them and make new pictures. These books are suitable for younger children and can stimulate an activity similar to "Sorting Shapes," but the focus would be only on line symmetry, not rotational symmetry.

Further explorations with mirrors for older children are suggested in *Another, Another, Another, and More* by Marion Walter (1975). This book comes with two mirrors that are to be taped together. The hinged mirrors can then be used on a page to reflect multiple images of a single image.

Another source of images that are seen in different ways upside down is *Topsys and Turvys* by Peter Newell (1964). Each page of this book contains a picture and a two-line poem. The first line is read while looking at the picture one way, and the second line is read after rotating the page, turning it upside down. Incredibly, the pictures change and the poems make sense! This book was originally published in 1894 and reprinted in 1902.

That Clarence Cowles leaned o'er the edge and hoisted Bertram out.



When Bertram Bowles fell off the dock, so loudly did he shout,

Charlie and the Great Glass Elevator

Roald Dahl (1972)

Summary of the Story (Chapters 13–19)

Mr. Wonka, owner of the most extraordinary chocolate factory, has invented a pill called Wonka-vite. Each pill makes the one who swallows it younger by exactly 20 years. Three old people, aged 81 years, 80 years and three months, and 78 years, each take four of these pills. The two older people become babies, and the youngest disappears and ends up in Minusland. The youngest is rescued from Minusland by being sprayed with Vita-wonk but gets an overdose and ages to 358 years. She is finally given enough Wonka-vite pills to return her to her original age. The others, who are babies, are returned to their normal ages by being given drops of Vita-wonk that each add 20 years to their ages.

Classroom Sketch (Grades 4–6)

Negative numbers

1. If possible, read these chapters of the book in parts, over several days. At the point where the old people have just taken the Wonka-vite pills, ask children to discuss what will happen to people of various ages, for example, 100, 95, 82, and 60. Draw a number line (0 to 100 or so) on which to represent the ages to help children visualize the subtraction. Elicit the idea of Minusland.
2. Continue reading the story until Mr. Wonka explains Minusland. Expand the number line to include negative integers. Ask children what can be done to get the unfortunate woman back from Minusland. Some may suggest another type of pill that would add a certain number of years. Continue reading until the woman has had too large a dose of Vita-wonk and let the children compute the necessary dosage of Wonka-vite to return the 358-year-old woman to the age of 78.
3. As a follow-up to this story, invent some other types of pills that change one's age by adding or subtracting a certain number of years. Let children work out problems of this type on a number line, perhaps seeing what happens if they take various combinations of each type of pill.

Other Books

Many books that prepare children for mathematical concepts are found in the *Wonderful World of Mathematics: A Critically Annotated List of Children's*

Books in Mathematics by Diane Theissen and Margaret Matthias (1992). More general listings of children's literature, such as *A to Zoo, Subject Access to Children's Picture Books* by Carolyn W. Lima and John A. Lima (1989), will furnish more ideas under the heading "Concepts."

Sequence

Many books for young children have strong repeating verbal patterns and a simple sequence of events to be put in order. A teacher can take images from such books, mix them up, and then encourage children to reconstruct the pattern of the story. These activities can precede experiences with forming and continuing more abstract visual patterns. The following books can be used in this way:

The Little Red Hen (Grades Pre-K–1). This book by Paul Galdone (1973) can be used in this activity. The classic story of ***The Gingerbread Man*** in any version is another example.

Comparison of Measurements

Many concept books involve big and small and sizes in between. A classic story is any version of ***Goldilocks and the Three Bears.*** This story lends itself especially well to a manipulative activity if you assemble pictures of all the items in the story in three sizes so that children can retell the story and decide who gets which bowl, spoon, chair, and bed. Two important ideas to introduce are that size is relative and that bigger is not necessarily better. Here are two other books that deal with these ideas:

Too Much Noise. A man finds his country house much too noisy in this book by Ann McGovern (1967), and he is advised to get a chicken to help. When that doesn't help, he is told to get a cat. The pattern is repeated, and his house gets noisier and noisier with more and more occupants, until finally, when he complains, he is advised to get rid of all of them. Now the house, which is just the same as it was originally, seems blissfully quiet! This story can be extended to the idea that what is loud in one context is quiet in another, what is big in one context is small in another, and what is heavy in one context is light in another. Physical experiences with sounds, shades of darkness on paper, weights, and so on, can reinforce this idea.

The King's Flower. A king is obsessed with having everything as big as possible to the point of absurdity in this book by Mitsumasa Anno (1970). Finally he sees a perfectly normal-sized flower, which will not grow to be big, and sees how beautiful it is.

Number

Counting books abound, but many are dull and didactic. Most can be used as in chapter 3, to motivate children to create their own counting books. A few are classics, and children do not seem to tire of them. Counting books are often read to children who are too young to have developed much understanding of number, and the books are expected to play a role in preparing for number concepts. Below are three that might be used in this context, with some thoughts on the reasons for their appeal—namely, a good story line, humor, challenge, a tactile quality, and an especially warm and cozy environment.

The Very Hungry Caterpillar. In this book by Eric Carle (1989), a caterpillar hatches from an egg and begins to eat—one piece of fruit on the first day, two pieces on the second day, up to the fifth day. On the sixth day, he eats an enormous amount of food; then on the seventh day, he has to rest to recover. Finally he turns into a butterfly. This book has a tactile

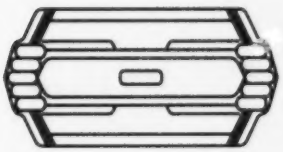
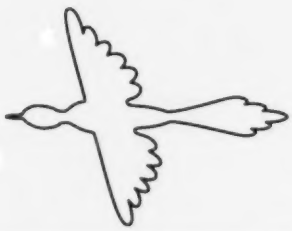
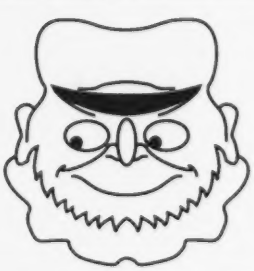
BEST COPY AVAILABLE

quality (children can put their fingers on little holes punched in the fruit to show what the caterpillar ate), which makes it appropriate for very young children who are not yet counting rationally. Yet the attractiveness of the pictures and the humor of the story keep older children and adults interested. To expand on the tactile quality, have children sort cutout pictures of fruit and act out the story with them.

Bears on Wheels. In a humorous story by Stan and Janice Berenstain (1969), various numbers of bears end up on various numbers of wheels (unicycles, bicycles, tricycles, cars). Counting is encouraged and amusing and a bit more complex than usual in a counting book. It could be extended by drawing pictures of different numbers of bears on different numbers of wheels.

Ten, Nine, Eight. This book by Molly Bang (1983) counts backward, which is a useful and sometimes neglected counting skill, and also embeds the counting in a comforting going-to-bed ritual. Perhaps this book symbolizes how comforting rote counting can be to young children. Many wonderful folk songs and lullabies are based on counting patterns.

CARDS FOR SYMMETRY EXPLANATIONS

		Reflections
69	191	40
A	B	C
F	N	O
WOW	NOON	

6

DEVELOP A CONCEPT OR SKILL

Books described in this chapter are suitable for developing mathematics concepts or skills that have already been experienced by children in an informal way and that now can be formalized and analyzed. As stated in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989), "emphasizing mathematical concepts and relationships means devoting substantial time to the development of understandings. It also means relating this knowledge to the learning of skills by establishing relationships between the conceptual and procedural aspects of tasks" (p. 17). Books suitable for establishing these relationships should provide a context that allows (1) interpretation of the mathematical idea by children with concrete or visual materials, (2) use of mathematical vocabulary, symbolism, and procedures in connection with the interpretation, and (3) natural adaptation of the context to provide varied examples and the opportunity for children to create their own examples.

The first example, *So Many Cats* (de Regniers 1985), tells a story involving addition. For children to understand fully the connection of the story with the operation of addition as seen in other contexts, they should previously have had experiences with combining collections of objects. The second example, *How Big Is a Foot?* (Myllar 1962), illustrates the measurement of length with nonstandard units and the need for a standard unit. The images will mean more to children if they have previously had experiences with measuring length by placing blocks, or other objects of uniform length, end to end next to an object to be measured.

These books can meet the needs of children at different levels in different ways. For example, if one of the books is read to an entire class, some children may follow the story completely, relying on mental images or the pictures in the book, whereas others may need to act out parts of the story with tangible objects to understand fully the mathematical concepts. The books suggested in chapter 5 can be adapted to the type of use suggested in this chapter by including more formal language. Also, most books suggested in chapter 7 can be adapted to fit this chapter by adding appropriate manipulative materials to act out the story.

So Many Cats!

Beatrice Schenk de Regniers (1985)
Illustrated by Ellen Weiss

Summary of the Story

This book describes how a family collected a dozen cats. They started with one, but then cats arrived in groups of one, two, three, or four, and none could be resisted. After each new arrival, the cats are counted again. The book ends with yet another cat peering in the window....

Classroom Sketch (Grades K–2)

Addition

1. Read the book, letting the children count the pictures in the book each time new cats appear. (If it is difficult for all children to see the pictures, you might have children draw the new cats on a chalkboard for other children to count.)
2. Reread the book, but this time have children use counters to act out the additions and decide how many cats there are each time. If appropriate, have a child write the corresponding addition sentence as each new cat or cats arrive (for example, $6 + 2 = 8$).
3. Have children take turns telling their own stories, real or imaginary, about how they acquired pets. (First I had a frog, then two alligators followed me home....) Other children can act out these stories using counters and write corresponding addition sentences.
4. To develop mental arithmetic, use the context of the story to invent new stories about cats that come or leave and ask the children to figure out in their heads how many cats there are in the new story. At first, concentrate on stories where just one or two cats arrive or leave.

Follow-up: Small-Group Activity

Make about three sets of cat counting cards from activity sheet 6.1. Each set contains six cards, with pictures of one to six cats on each card. Children can work in a group, taking turns drawing a card and telling a story about how many cats there were, how many new cats arrived, and how many cats there are now in all.

These cat counting cards can also be used to develop estimation skills. Show a number of the cards, totaling too many cats to count directly, and ask children to estimate how many cats there are in all. Have them discuss their strategies; for example, they might roughly make groups of ten cats and count by tens.

Note: This book can also be used in the same way as *Millions of Cats*, described in chapter 2. The pictures on the cat counting cards are drawn from the attribute set on activity sheets 2.8 and 2.9.

How Big Is a Foot?

Rolf Myllar (1962)

Summary of the Story

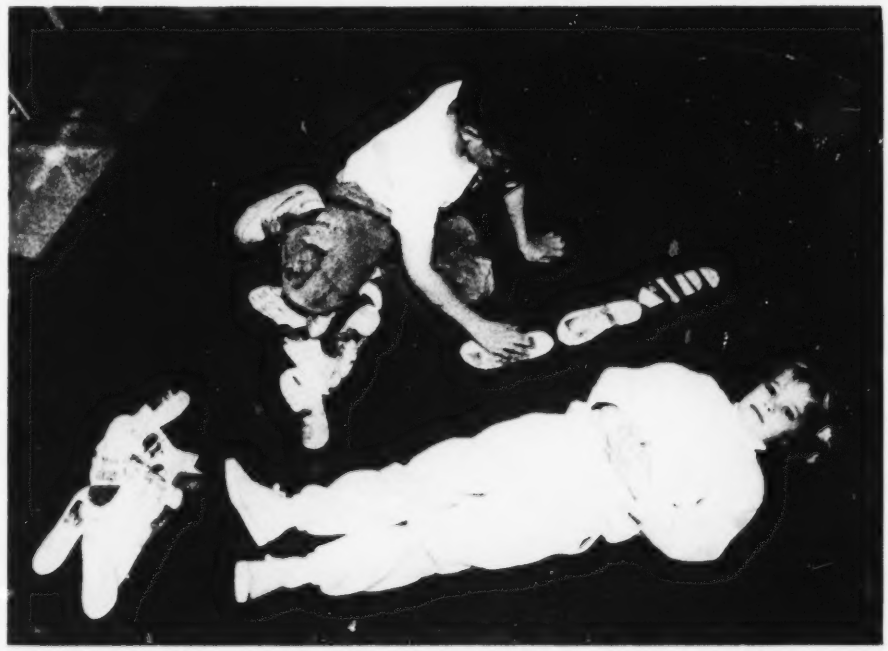
A king wants a bed made to fit his wife, the queen. Measurements for the bed are made using the king's foot. The apprentice carpenter, who is responsible for making the bed, gets into trouble because the measurements made with his small foot do not correspond to those made with the king's foot, and the bed he makes is too small. The apprentice finally figures out why, and a marble copy of the king's foot is made to serve as a standard unit of length.

Note: Be alert to the fact that the only woman in this story is the queen, and all she does is lie down to have the bed measured for her. The potential gender stereotyping of this book can be avoided by calling the apprentice "she."

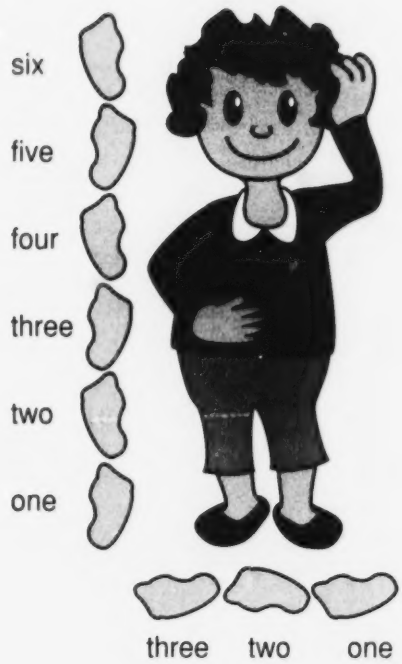
Classroom Sketch (Grades 1–3)

Measurement of length

1. Read the story to the children. Pose the question "How big would a bed for one of you need to be?" Have one child lie down on the floor and other children measure him or her, using their feet as nonstandard measurements. Two very different sizes of feet should be used. *Why do the measurements differ?* In this way, develop the need for, and the concept of, a standard unit of measure. Discuss when an individual's feet are suitable for measuring and when they are inadequate (i.e., for smaller or greater lengths).
2. Children can make their own foot measures by tracing their feet on a newspape, and cutting through many layers. These footprints can be used to measure objects in the classroom. Ask children to estimate how many footprints will be needed before they try. (Note that calling the units "footprints" avoids confusion with the English measure called a foot.)
3. Collect a footprint from each child and arrange the footprints in order of length. Discuss how the class could select a "typical" footprint to serve as the class's standard unit. They might pick the length in the middle (i.e., the median). Have them use this class standard to estimate and measure distances in



BEST COPY AVAILABLE



the school. To emphasize that different units give different measures, also cut out copies of the "king's" footprint (perhaps the principal could be invited to contribute his or her footprint). Children can work in pairs to complete the record on activity sheet 6.2, where each child is responsible for measuring with one of the two footprints.

Bulletin-Board Display

A challenge question for the hallway. At the start of the week, display a poster showing one footprint and asking passersby to estimate how many would fit up the wall, from floor to ceiling. At the end of the week, children can actually tape the footprints up the wall.

Follow-up: Homework Assignment

Have the children take the cutout footprints home to measure their own beds. Probably their beds are larger than they need to be. Would each child be comfortable in a bed that is three of their footprints wide and six of their footprints long (the dimensions used in the story)?

Extension: Investigation of Ratio (Grades 2–6)

This book provides a context in which to investigate ratio. Pose questions such as this: *If, when measuring a desk, a child's foot yields a measure of six footprints and an adult's foot yields a measure of three footprints, can you predict how many of the child's footprints will be needed to measure a table that is five adult footprints long?*

Extension: Investigation of the Relationship between Two Sets of Data (Grades 2–6)

If the footprints of the class are arranged in order of size , a natural question to ask is whether the tallest children have the longest footprint and the shortest children have the shortest footprint. Children might explore ways to represent this relationship. They might make a list of children in order of height, below that arrange the footprints in order of length, and draw the matches. This question would be a good context for introducing a scattergraph, in which children graph the set of ordered pairs formed by their height in centimeters and the length of their footprint in centimeters.

Other Books

Many classroom activities described in earlier chapters are useful for developing a mathematics skill or concept; however, in most of the earlier examples, the books suggest a wider range of uses as well. The books listed in this chapter seem suitable for developing a quite specific skill. Good places to look for more examples of such stories are *The Wonderful World of Mathematics: A Critically Annotated List of Children's Books in Mathematics* by Diane Thiessen and Margaret Matthias (1992), and the supple mentary materials that accompany textbooks from other publishers that attempt to embody the vision of the *Standards*.

Number Sense and Numeration

Ed Emberley's Picture Pie (Grades 2–4). This book by Ed Emberley (1984) is described in chapter 3. In the classroom sketch given there, children create patterns from fractional parts. Once these patterns are created, their use might fit into this chapter to develop understanding of equivalent fractions and mixed numbers.

If You Made a Million. This book by David M. Schwartz (1989) resembles *How Much Is a Million?* (Schwartz 1985), described in Chapter 7, but focuses on money. The explicit consideration of the relationships among amounts (i.e., visual images of a quarter or a dollar in terms of dimes, nickels, and cents) makes the book suitable for developing number sense.

Operations on Numbers

The Doorbell Rang. Written by Pat Hutchins (1987), this book is described in chapter 5. With the modifications described at the end of the classroom sketch, it could fit in this chapter.

Anno's Mysterious Multiplying Jar (Grades 4–6). Factorials are introduced and explained visually. This book by Masaichiro Anno and Mitsumasa Anno (1983) would be suitable for introducing the factorial notation as soon as students understand the meaning of multiplication.

Time

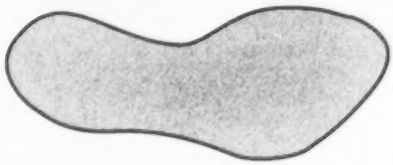
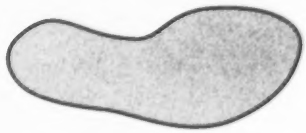
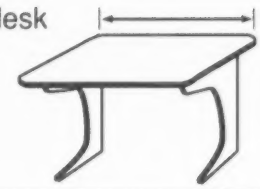
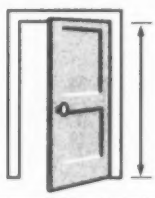
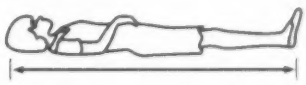
All in a Day. This book by Mitsumasa Anno (1986) presents the concurrent events of a day in several children's lives in eight different countries and an "uninhabited island" (with illustrations by nine renowned artists, each with a characteristic style). Each event is labeled with its time, and readers rapidly become aware that the time is different in different time zones at a given moment. A note at the end of the book discusses world time zones and how time is measured in different places.

Clocks and More Clocks (Grades 1–2). A man thinks that his clocks, which are on different floors of his house, are defective because as he goes from one floor to another, the clockfaces show different times (because of the time it takes him to go from one clock to another). Finally he learns that the clocks are all correct by carrying a clock with him as he moves around the house. For children with some experiences with clocks, this story by Pat Hutchins (1973) can clarify thinking about the measurement of time and can also lead to questions about how long it actually takes to go from one part of a house to another.

Activity Sheet 6.1
Cat Counting Cards

Activity Sheet 6.2

MEASURING WITH FOOTPRINTS

	King's footprint		Apprentice's footprint	
				
	Estimate	Measure	Estimate	Measure
Length of desk 				
Height of door 				
Length of _____ (A friend) 				
(Choose your own) _____				
(Choose your own) _____				

What do you notice?

7

PROVIDE A CONTEXT FOR REVIEW

The *Professional Standards for Teaching Mathematics* (NCTM 1991) states that one consideration in the selection of "worthwhile mathematical tasks" is "to assess what the task conveys about what is entailed in doing mathematics." A goal for the selection of tasks that develop "appropriate skill and automaticity" is to "create contexts that foster skill development even as students engage in problem solving and reasoning" (p. 26). Some books naturally provide such contexts to review or practice mathematics skills. There is a mathematical question to be answered, and the context suggests that students create more mathematical questions. Many such books involve earning and spending money. These are useful for children who already are familiar with the computational skills involved and need practice. Such stories could also fit into the previous chapter if the children are not yet familiar with the computational algorithms and need a manipulative (such as coins) to act them out.

In *Arthur's Funny Money* (Hoban 1981) and also in *The 500 Hats of Bartholomew Cubbins* (Dr. Seuss 1966), the entire story centers on a running computation. Another example, *How Much Is a Million?* (Schwartz 1985), is not really a story but is useful because it asks questions directly, which provides practice with computational skills in a problem-solving context. When you read these books in the classroom, it is a challenge to keep students as actively involved as possible; do not let them become passive listeners. It may be appropriate to stop frequently and wait until all children have had a chance to think and respond to questions about the story.

If you are oriented toward a problem-solving approach to mathematics, it is likely that your use of a book in this chapter will infringe on the types of uses in chapters 3 and 4. Even if the book asks a fairly obvious, direct question, you may be tempted to extend it by having your students write their own questions or even alter the context and write a similar story.

Arthur's Funny Money

Lillian Hoban (1981)

Summary of the Story

In this "I Can Read" book, Arthur wants a certain amount of money to buy a T-shirt. His piggy bank does not contain enough money; so he decides to earn money by washing bicycles. He has initial expenses of soap and cleaning pads. Various decisions confront him as he starts washing the bicycles, such as what to charge for a small tricycle and how to advertise. Eventually he needs more soap and finds that the price of soap has gone up but that the T-shirt he wants is on sale. He has not up to this point counted how much money he has. He finds that he has more than enough money to buy his shirt and uses the remaining money for some licorice twists for himself and his sister. The story involves the computation $3.78 – 0.53 – 0.27 + 0.42 + 0.34 + 0.36 + 0.33 – 4.25 = $0.18. The final question is, How many licorice sticks, costing five cents each, can he buy?

Classroom Sketch (Grades 2–3)

Two-digit addition and subtraction

1. Read the book to the children. Stop at the point where Arthur goes into the store, finds out that the T-shirt costs $4.25, and is asked by the saleslady if he has enough money. " 'I don't know,' said Arthur." Ask the children if they know. They probably won't, unless you asked them to keep track of the money. Suggest that you read it again and that this time the children keep a running record.
2. Arrange the children in groups so that they can confirm each other's interpretation of each step of the story. Read the story again, stop after each transaction to allow children time to work out how much money Arthur has at that point, and continue to the end. If appropriate, ask them to figure out how much more he needs at each point to buy his T-shirt.

Note: Younger children might have a collection of play money with which to act out the story. This use of the book would fit into the previous chapter. For older children just paper and pencil would be appropriate.

Follow-up: Group Planning and Writing

Arrange the children in groups. Ask each group to think of a way in which they might make money and to list projected expenses and earnings. Have each group write a story similar to this book, read it to the class, and let the other children follow along and act out the story.

How Much Is a Million?

David M. Schwartz (1985)
Illustrated by Steven Kellogg

Summary of the Book

Many images of a million are given, including how long it would take to count to a million, what a million dots on paper would look like, and how far 1 million children standing on each other's shoulders would reach. Similar images are provided for a billion and a trillion. At the end of the book is an explanation of how the author arrived at his estimates.

Classroom Sketch (Grades 3–6)

Large numbers
Multiplication
Estimation strategies

1. Start to read the book to the class but stop at appropriate places to ask the students to anticipate the answers. Some good pages for this are those concerned with how long it would take to count to 1 million, to 1 billion, and to 1 trillion. For example, show the top of the page that starts, "If you counted aloud from one to one million, it would take you ...," but cover the answer. First ask for rough estimates, then ask students to work in groups to develop specific estimation strategies and to formulate a more careful estimate.
2. Show the author's estimate for how long it would take to count to 1 million. This may be much longer than students' estimates because their initial thought might be to allow one second for each number, but in fact, most five-digit numbers take more than a second to say aloud. Ask students why their estimates differ from the author's or, if they are close, if they think they used the same strategy the author did.
3. Test the author's estimate of the average time to say a five-digit number. Write a list of five-digit numbers on the chalkboard and time various children as they read them aloud. If desired, average their reading times. Discuss how many numbers less than 1 million are five-digit, four-digit, three-digit, two-digit, and one-digit numbers. Do students agree with the author's estimates now?
4. Repeat the procedure above for the pages, asking how long it would take to count aloud to 1 billion and to 1 trillion.

Follow-up: Models for 1 Million

The book suggests many models for 1 million. Challenge groups of students to find their own model of 1 million. They might think of grains of rice (see chapter 4) or seconds elapsed in a year or pages in books on library shelves.

A particularly illuminating model can be constructed using cubic centimeters (white Cuisenaire rods). A stick made from ten of these cubes (an orange Cuisenaire rod) represents 10, and a square made from ten orange rods represents 100. Ten of these squares stacked into a cubic decimeter represent 1000 cubic centimeters. Materials like these are available commercially. If students then imagine repeating this process using the 1000-cube in the place of the original cubic centimeter, they can visualize models for 10 000, 100 000, and 1 000 000. Approximate physical models can be constructed from cardboard half-gallon milk or juice cartons. Students can collect these at home and measure 10 cm up three sides and 20 cm up the fourth side, as shown in figure 7.1. The extra 10-cm piece on the fourth side is folded over and taped to make a cube that is approximately a cubic decimeter. These cubes can be taped together to make a "giant orange Cuisenaire rod," with a volume of 10 000 cubic centimeters. If students persevere, they can collect enough cubes to make the giant square, with a volume of 100 000 cubic centimeters. To visualize one million, they could build a cubic meter from metersticks taped together and imagine it filled entirely with cubic centimeters.

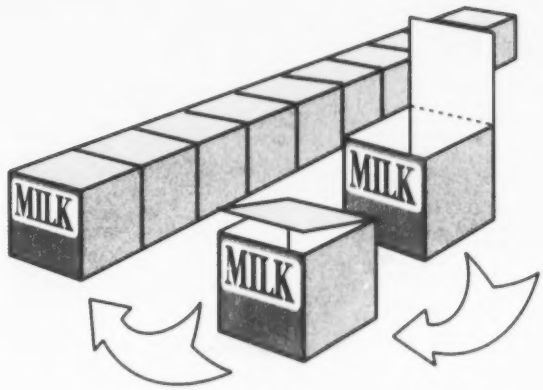


Fig. 7.1

Other Books

The Wonderful World of Mathematics: A Critically Annotated List of Children's Books in Mathematics by Diane Thiessen and Margaret Matthias (1992) lists further examples of books related to specific mathematics topics. Below are two types of stories that lend themselves especially well to the practice of computational skills.

Earning and Spending Money

Several books have a story line involving money being earned or spent. One of the most amusing is the following:

Alexander, Who Used to Be Rich Last Sunday (Grades K–2). In this book by Judith Viorst (1980), illustrated by Ray Cruz, a boy starts with one dollar and spends it a little at a time. The book can be used as in the classroom sketch for *Arthur's Funny Money* (Hoban 1981), at a more elementary level.

Accumulation or Loss of Objects

The next examples involve the accumulation of other objects; in the first, it is hats, in the second, bandages, and finally, birds. These books show that when written by masters of children's literature, routine arithmetic procedure can be built into an engrossing and amusing context.

The 500 Hats of Bartholomew Cubbins (Grades 1–2). Bartholomew has a terrible problem in this book by Dr. Seuss (1966). When he takes off his old hat in honor of the passing king, a new one immediately appears. This is considered a great offense by the king. Bartholomew keeps removing hats, and new ones keep appearing. He is arrested, and all sorts of methods are tried to remove the hats. He is threatened with beheading (but it is against the rules to be beheaded with a hat on, so he is saved). A running count is kept of the hats (which at times leads to addition problems). At the last moment, as Bartholomew is about to be pushed off a high tower, the king takes a fancy to the wonderfully elaborate 500th hat, which he removes from Bartholomew and buys, and no new hat appears. As you read the story, stop at points where an addition example arises and ask children to keep the running total.

Where the Sidewalk Ends (Grades 2–3). The poem "Band-Aids," about a child who has an amazing number of adhesive bandages all over him, can be used to provide practice in mental addition. Read the poem slowly and have children keep a running total in their heads. The addition in the poem is 1 + 1 + 1 + 2 + 3 + 9 + 2 + 1 + 1 + 1 + 4 + 5 + 1 + 1 + 1 + 35, which equals 69. Another poem in this collection by Shel Silverstein (1974) that can be used to review the understanding of values of coins is "Smart," in which a child boasts of trading one dollar bill for two quarters, then those for three dimes, and those for four nickels, and those for five pennies—all the while feeling very pleased with himself. After each verse, children could mentally compute how much money the boy has then and how much he has lost from the original dollar.

A Book of Nonsense by Edward Lear, first published in 1846 (Jackson 1951), contains the following limerick (p. 3) that can be used in the same way as "Band-Aids," but with younger children:

> There was an Old Man with a beard,
> Who said, 'It is just as I feared!—
> Two Owls and a Hen,
> Four Larks and a Wren,
> Have all built their nests in my beard!

Children can be encouraged to make up similar nonsense rhymes with arithmetical puzzles in them to practice computational skills.

8

GENERAL APPROACHES

The preceding chapters concerned using particular books or types of books to enhance children's mathematical experiences. In this chapter, we will examine some ways in which mathematical thinking can be applied in the classroom experience of children's literature in general.

One of the underlying assumptions of *Professional Standards for Teaching Mathematics* (NCTM 1991) is that "teachers are key figures in changing the ways in which mathematics is taught and learned in schools" (p. 2). Teachers play a vital role in the recommended shift "toward connecting mathematics, its ideas, and its applications—away from treating mathematics as a body of isolated concepts and procedures" (p. 3). The teaching suggestions in this chapter rely on the teacher to incorporate mathematical language and thinking into students' experience of literature in general, beyond the context of a mathematics lesson.

Questions that relate to mathematics can be asked when selecting a book to read at any level. For example, consider the following:

- What type of book shall we read? (*classification*)
- How many in the class have already read this book? (*counting*)
- Which of these three books would the class prefer? (*comparison of number*)
- Shall we vote? (Represent the vote on a *graph* made with magnetic name tags over the preferred book.)
- Do we have time to read this book? About how long will it take? (*estimation and measurement of time*)
- There are six school days before vacation, and we seem to be reading about nine pages a day. Can we finish? (*computation*)
- We were on page 98 and we will have time for about five pages. How far will we get? (*computation*)
- How are the pages numbered? Can you tell on which side, right or left, page 24 will be? Page 31? (*even numbers* and *odd numbers*)
- Why are there sometimes roman numerals on pages at the start? (*numeration*)
- How are books made? (Discuss how the fold-up book in chapter 2 is made and how the same principle applies to larger books.) In a bound book, how many pages are in the clumps of pages that are sewn together? Why? (*number patterns*)

Of course these questions should not predominate in the part of the school day devoted to reading children's literature, but by asking such questions with a "light touch," you can show children that mathematical tools can be helpful in many parts of their lives.

In the next section, Elizabeth Phillips, a first-grade teacher at P.S. 321 in Brooklyn, New York, writes about how mathematics was woven throughout her class's study of the works of Tomie dePaola. As you read this article, think of how similar ideas can be developed during the study of other authors.

Integrating Mathematics into an Author Study

Elizabeth Phillips

By studying one author of children's books in depth, children gain a new appreciation of literature, learn strategies for independently choosing reading material, and see the reading-writing connection come alive as they compare themselves as writers to the writer whose life and work they are studying. Teachers who have used author studies in their classrooms have found them to be wonderful vehicles for developing a sense of community in the classroom. And partly because author studies depend on comparing and contrasting a number of books, these studies offer many opportunities for integrating mathematics and children's literature.

I will outline here an author study of Tomie dePaola that is particularly rich in mathematics connections and that I carried out in my first-grade class. Many of the activities could be adapted to other author studies, and many are appropriate at other grade levels. Few teachers will be able to use all the

activities listed; they should pick and choose from the list. And since dePaola is such a prolific author, many more activities could be combined with his books—and many more books than I discuss are available.

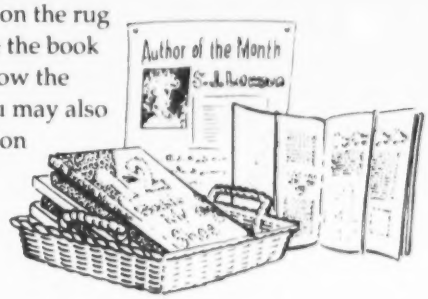
Author Study Introduction

1. Make a poster featuring the "Author of the Month," Tomie dePaola, that includes book titles, biographical information, and if possible, a picture of the author.

   Math Connection: Include dates on the poster; this may lead to informal discussion about the author's age.

2. Gather as many Tomie dePaola books as you can from your school and public library, from your own and your colleagues' collections, and from the collections of the children in your class. Many of his books are available in paperback, some from book clubs. Ideally, you will have at least ten books. Keep all books by the author in a special basket in the library area with the sign "Books by Our Author of the Month."

3. Begin the study with a class meeting on the rug and introduce the book basket and show the titles in it. You may also want to mention briefly some biographical information about dePaola.



   Math Connection: If some of dePaola's books are well known to the class, you may want to make a quick graph of the books with which the children in the class are familiar.

   Math Connection: Ask the children how many books they think Tomie dePaola has written and illustrated. Make a graph of their guesses, using

BEST COPY AVAILABLE

grouped numbers, such as 1–19, 20–39, and so on, ending with 100+. Discuss what it means to say "100+." *Why do we add the "+"?* Children may be surprised to hear that dePaola is the author of more than 40 books and the illustrator of more than 100 others.

Math Connection: DePaola began his career in 1965. *How many years has he been writing?* For sophisticated second graders, you might want to ask how many books he would have had to write or illustrate a year to have completed 140+ books by 1989. *Do you think he actually wrote the same number of books every year?*

Folktales

1. Read *Strega Nona* (1975) to the class and discuss it. Share with the class the facts that dePaola has an Italian grandmother, that he loves to cook, and that he loves folktales. He says, "If you read enough folktales, you're going to find out everything.... The real reason I write them though is that I loved them when I was growing up."

   Math Connection: Follow reading the book with a "choice time" activity of making (and then eating) pasta. Have the recipe on chart paper. Depending on the grade level, talk about doubling or halving the recipe; predict how many cups of cooked pasta you can make from one cup of uncooked pasta. Many variations are possible here, including the use of nonstandard or metric units for measurement in cooking.

   Math Connection: Introduce pasta as a mathematics manipulative and use it for many different activities. (See pp. 17–20) Use different shapes or colors of pasta (which can be dyed with food coloring and alcohol—mix and shake with pasta in a plastic bag) for patterning, as a counting manipulative material, for sorting activities, and so on.

2. Read two other Strega Nona books—*Big Anthony and the Magic Ring* (1979) and *Strega Nona's Magic Lessons* (1982)—and discuss similarities and differences from book to book. If you have multiple copies of some of these books, you may want to divide into small groups after an all-class reading so that children can look at the illustrations closely and discuss the book. Depending on the grade level, these small-group discussions can be free flowing or directed (e.g., *Find your favorite part and discuss it; See what you notice in the illustrations*; etc.) and might at times include writing and drawing activities as well.

   Math Connection: Graph the children's preferences of Strega Nona books.

3. Read other folktales by dePaola, such as *The Legend of the Blue Bonnet* (1983), *The Clown of God* (1978), or *Charlie Needs a Cloak* (1973). Discuss folktales and what it means to "retell" a folktale. Expand on this theme by finding other versions of some of the tales dePaola has retold. Children who love *Strega Nona* will be excited to hear *The Magic Porridge Pot* by Paul Galdone (1976) or *The Magic Cooking Pot* by Faith Towles (1975). The idea of retelling can be integrated into writing time, and children can write their own versions of folktales.

   Math Connection: Introduce children to the copyright page and explain how they can find out when the books were published. Put books in order by publishing date; develop related questions appropriate for each grade level—for example, How much time elapsed between the publication of *Strega Nona* and *The Clown of God?* Had the children in your class been born when these books were published? Continue to return to the copyright information throughout the study.

Realistic Fiction

1. Tomie dePaola has written a number of realistic-fiction titles that are autobiographical; many deal with sensitive issues, such as death. Some of my favorites in this category are *Nana Upstairs and Nana Downstairs* (1973), *Now One Foot, Now the Other* (1981), *The Art Lesson* (1989), *Watch Out for the Chicken Feet in Your Soup* (1974), and *Oliver Button Is a Sissy* (1979). Begin reading these books and discussing them.

   Math Connection: At this point—or possibly later—you may want to begin a chart that categorizes dePaola's books. Then as you read each new one, the children can decide into which category it falls. You will probably want to let the children generate the ideas for the

categories, and what you come up with will depend on which dePaola books you have.

2. Throughout the author study you will want to introduce small bits of biographical information. During this part of the study, this information is particularly important, and as you talk about dePaola's life, you may want to point out how events in his life are translated into his books.

   Math Connection: Begin a time line of dePaola's life. Include the publication dates of the books you are studying in class. Depending on the grade level, activities can include figuring out how many years ago dePaola wrote *Strega Nona*,how many books he wrote in a ten-year period, and so on. Again depending on the level, you might introduce the concept of a decade. Above or below the timeline, include information that makes the dates meaningful to the children; for example, *The Legend of the Blue Bonnet* was published in 1983—that was the birth date of many of my first graders at the time of this author study.

3. Productive minilessons for writing time can be centered on the idea that dePaola uses events and feelings from his life so effectively in his fiction.

4. Many of Tomi dePaola's realistic-fiction books lend themselves to discussion and activities related to the passage and measurement of time. For example, in *The Art Lesson*, the concepts of the days of the week and "next month" are introduced. Depending on the particular story, different types of calendar mathematics can fit into an author study.

5. Two recurring themes in dePaola's literature are food and grandparents. Each theme could be explored with various spin-off activities.

   Math Connection: If your class is familiar with using Venn diagrams to classify by attributes, you can use these diagrams effectively to categorize some of dePaola's books. For example, one loop might be "books where food is important" and the other "books where grandparents are important." In this siituation, *Watch Out for the Chicken Feet in Your Soup* would be in the overlapping loops (see fig. 8.1).

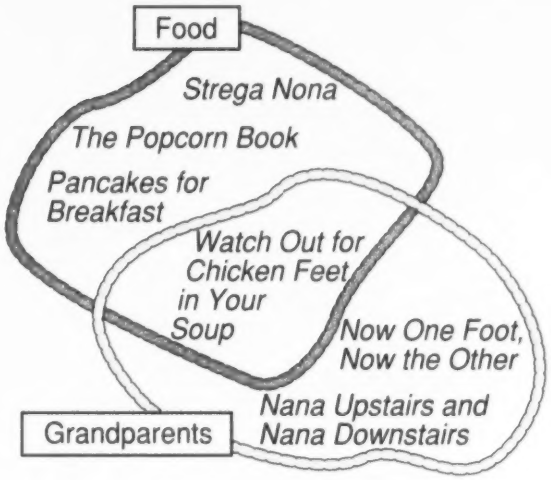


Fig. 8.1

Information Books

Examples of dePaola's information books are *The Popcorn Book* (1978), *The Quicksand Book* (1977), *The Cloud Book* (1975), and *The Kid's Cat Book* (1979).

1. Read *The Popcorn Book*. Have the children make a chart of what they learned from the book. *How is this book different from other dePaola books? In what ways is it similar?* Share the fact that dePaola loves to eat popcorn.

   Math Connection: Make popcorn. Predict how many unpopped kernels you need to make one liter of popcorn.

BEST COPY AVAILABLE

Math Connection: Compare how popcorn expands during cooking with how pasta expands. How would you determine this? Have children brainstorm about ways to measure. Does the weight of the popcorn (and pasta) increase when cooked?

Math Connection: Fill two small jars of the same size—one with popcorn kernels and the other with popped pieces. Have children estimate the number of kernels and pieces in each jar. Graph the estimates. Count the actual pieces and stop when you have counted about half. Then allow children to revise their estimates and explain why. Graph the results.

2. Read *The Quicksand Book*. Discuss with the class what they learned from the book. *How is the book similar to* The Popcorn Book? *What makes it recognizable as a dePaola book?*

   Math Connection: Make quicksand using the recipe in the book.

   Math Connection: Perform a series of weight activities with sand, using nonstandard measurement first (for example—one liter of sand weighs about the same as. . .). Compare the weights of a liter of sand and a liter of popcorn. Depending on the grade level, you could then move to measuring with standard units.

3. Read *The Cloud Book* with the children and discuss it. Go outside and look at clouds. Draw cloud pictures with chalk and blue paper.

4. Read *The Kid's Cat Book* and discuss it; compare it to other dePaola information books.

   Math Connection: Survey the children in the class. *Do you have a cat?* or *What kind of a pet do you have?*

   Math Connection: Use the information about the Chinese New Year on the last page of the book. Have the children try to figure out when the next Year of the Cat will be if the last ones were in (as dePaola says) 1963, 1975, and 1987. One note on this—according to other sources, these years were not the Year of the Cat but the Year of the Rabbit. Did dePaola (and his editors) make a mistake? This might be an interesting question for students to research.

5. Relate nonfiction books to the children's writing. Can the children write information books? *How would you choose a topic? How does dePaola make his information books interesting? How can you?*

More Books

1. Show the class *Pancakes for Breakfast* (1978), a wordless picture book. Talk about how this book allows the children to make up their own story. About a week later, have a small group work during choice time to dictate and then illustrate the students' version of the story.

   Math Connection: Make pancakes using the recipe in the book. Depending on the grade level, you could double or halve the recipe. Predict how many pancakes you will get from the recipe. You might convert the recipe to nonstandard or metric units of measurement.

DePaola has other wordless picture books, including *The Hunter and the Animals* (1981) and *Sing, Pierrot, Sing* (1983), that you may want to read and compare to *Pancakes for Breakfast*.

2. Read *Michael Bird-Boy* (1975) and discuss the story. Focus on the illustrations. Do they remind you of any of dePaola's other books? (This comparison could be made with any of his books.) Look at the illustrations in a number of dePaola books; he uses certain items—hearts, cats, white birds—frequently in his illustrations.

   Math Connection: Divide the children into small groups. Have each group work with a few books and graph or tally the number of hearts, white birds, and cats they see in the books.

3. The story *Andy That's My Name* (1973), in which children use blocks that are letters to make many words, can lead to wonderful language arts activities, such as making as many words as possible from the letters in the student's name or in Tomie dePaola's name.

   Math Connection: In how many different ways can you write a string of letters from A, N, D, and Y? (No repeats are allowed.) *How many ways are there with one letter? With two letters? Three letters? Four letters? How many of these strings are words? Can you predict how many different letter strings there would be for a three-letter word? A five-letter word? What strategies could you use to figure out the answers?*

Culminating Activities

You could either use one of the activities above (such as making pancakes) as a culminating activity for this author study or do one or more of the following:

- Have children illustrate a scene from a favorite book.
- *Math Connection:* Graph your students' preferences for their favorite dePaola book.
- In a second-grade or third-grade class, have small groups of children practice reading one of the books; then go to a kindergarten class to read and share information about dePaola.
- *Math Connection:* Complete the time line.
- Have the class listen to a tape of an interview with Tomie dePaola (available from Trumpet Publications) and make a chart in two colors. Use one color to show facts we know about his life; under each fact about his life, write in another color where this information appears in his books.
- Have children choose a favorite character from a dePaola book and make a puppet of the character.
- *Math Connection:* Survey another class about which dePaola books they have read or heard about.
- Have a Tomie dePaola party featuring food from his books (popcorn, pasta) and decorations of hearts, cats, and white birds. A variation on this theme that includes a *Math Connection* is a Tomie dePaola day, on which all curriculum areas relate to dePaola in some way (mathematics with pasta or popcorn, spelling with "ANDY" words, etc.).

Additional Information

For biographical information on Tomie dePaola, see book jackets or contact his publishers. In addition, see the following journal articles about dePaola and his books:

Abrahamson, Richard F., and Marilyn Colvin. "Tomie dePaola: Children's Choice." *Reading Teacher* 33 (December 1979): 264–69.

Andersen, Dennis. "Tomie dePaola: Tough and Tender Storyteller." *Instructor* 89 (March 1980): 32–42.

dePaola, Tomie. "Children's Book Illustrators Play Favorites." *Wilson Library Bulletin* 52 (October 1977): 165–76.

___. "From an Artist/Author: Tomie dePaola." *Language Arts* 58 (April 1981): 408–10.

Helper, Susan I. "Tomie dePaola: Gift to Children." *Language Arts* 56 (March 1979): 296–301.

Kerstetter, Kristen. "The Books of Tomie dePaola." *The Web* (February 1983).

Other Author Studies

Most of the activities described in the Tomie de Paola author study can be adapted to the study of any other author. When choosing an author for your class, it is important to select one whose books you are excited about because you will convey that enthusiasm to your class. Ideally, the author you choose will have written many books to which you have access. Depending on the grade level and the availability of books, you can read the books aloud or have the children share their thoughts after reading the books on their own.

The following is a brief list of some authors whom teachers have found suitable for author studies; it is only a beginning, and there are many other possibilities. The grade levels noted are flexible. In particular, picture-book authors, such as Tomie de Paola, Vera B. Williams, and Chris Van Allsburgh, work in the upper grades as well as the lower grades.

Grades K–2	Grades 3–6
Frank Asch	Ann Cameron
Byrd Baylor	Carol Carrick
Marc Brown	Ruth Chew
Margaret Wise Brown	Beverley Cleary
Eric Carle	Roald Dahl
Donald Crewes	Jean Fritz
Dr. Seuss	Jean Craighead George
Rita Golden Gelman	Eloise Greenfield
Eric Hill	Virginia Hamilton
Tana Hoban	Joanna Hurwitz
Ann Jonas	Lois Lowry
Ezra Jack Keats	Patricia Maclachlan
Leo Lionni	Scott O'Dell
Arnold Lobel	Katherine Patterson
Cynthia Rylant	Gary Paulson
Chris Van Allsburgh	Louis Sachar
Vera B. Williams	Lawrence Yep

Many resources can help you plan an author study. Professional journals regularly feature interviews with authors that you can share with your class, as well as occasional articles on implementing author or illustrator studies. Among these journals are *The New Advocate, The Horn Book, Language Arts, The Web, Five Owls,* and *Booklinks.* In addition, the Trumpet and Scholastic book clubs both now feature authors each month on the inside cover of the teacher's book order.

Other good sources of biographical information on authors are the publisher's publicity department and book jackets. There are also several books that include write-ups on many authors, including three books by Teachers' Idea Press—*An Author a Month for Pennies* (1989), *Bookpeople I* (1990), and *Bookpeople II* (1990)—and *Meet the Authors and Illustrators*. And when you begin author studies, clip articles you find on authors and establish your own files.

The following reference list refers to the books mentioned in this section on the author study. See also the bibliography for teachers and the booklist for children.

References for Author Study

dePaola, Tomie. *Andy That's My Name*. Englewood Cliffs, N.J.: Prentice-Hall, 1975.

____. *The Art Lesson*. New York: G. P. Putnam's Sons, 1989.

____. *Big Anthony and the Magic Ring*. New York: Harcourt Brace Jovanovich, 1979.

____. *Charlie Needs a Cloak*. Englewood Cliffs, N.J.: Prentice-Hall, 1973.

____. *The Cloud Book*. New York: Holiday House, 1978.

____. *The Clown of God*. New York: Harcourt Brace Jovanovich, 1978.

____. *The Hunter and the Animals*. New York: Holiday House, 1978.

____. *The Kid's Cat Book*. New York: Holiday House, 1978.

____. *The Legend of the Blue Bonnet*. New York: G. P. Putnam's Sons, 1983.

____. *Michael Bird-Boy*. Englewood Cliffs, N.J.: Prentice-Hall, 1975.

____. *Nana Upstairs and Nana Downstairs*. New York: G. P. Putnam's Sons, 1973.

____. *Now One Foot, Now the Other*. New York: G. P. Putnam's Sons, 1981.

____. *Oliver Button Is a Sissy*. New York: Harcourt Brace Jovanovich, 1979.

____. *Pancakes for Breakfast*. New York: Harcourt Brace Jovanovich, 1978.

____. *The Popcorn Book*. New York: Holiday House, 1978.

____. *The Quicksand Book*. New York: Holiday House, 1977.

____. *Sing, Pierrot, Sing*. San Diego: Harcourt Brace Jovanovich, 1983.

____. *Strega Nona*. Englewood Cliffs, N.J.: Prentice-Hall, 1975.

____. *Strega Nona's Magic Lessons*. New York: Harcourt Brace Jovanovich, 1982.

____. *Watch Out for the Chicken Feet in Your Soup*. Englewood Cliffs, N.J.: Prentice-Hall, 1974.

Galdone, Paul. *The Magic Porridge Pot*. New York: Seabury Press, 1976.

Towles, Faith. *The Magic Cooking Pot*. Boston: Houghton Mifflin, 1975.

BOOKS FOR TEACHERS

Note: The books in this list include references cited in the text and books that contain more information on children's literature and how to use it to teach mathematics.

Baratta-Lorton, Mary. *Workjobs II, Number Activities for Early Childhood*. Palo Alto, Calif.: Addison-Wesley Publishing Co., 1979.

Beyer, Jinny. *Patchwork Patterns*. McLean, Va.: EPM Publications, 1979.

Cullinan, Bernice E., ed. *Children's Literature in the Reading Program*. Newark, Del.: International Reading Association, 1987.

Fuys, David, and Rosamond Welchman Tischler. *Teaching Mathematics in the Elementary School*. Boston,: Little, Brown & Co., 1979.

Gutcheon, Beth. *The Perfect Patchwork Primer*. New York: Penguin, 1973.

Haldale, J. B. S. "On Being the Right Size." In *The World of Mathematics*, edited by James R. Newman, vol. 2. Redmond, Wash.: Microsoft Press, 1988.

Harsh, Ann. "Teach Mathematics with Children's Literature." *Young Children* 42: 24–29.

Lima, Carolyn W., and John A. Lima. *A to Zoo, Subject Access to Children's Picture Bookss*. 3d ed. New York: R. R. Bowker, 1989.

Lipson, Eden Ross. *The New York Times Parent's Guide to the Best Books for Children*. New York: Random House, 1988.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: The Council, 1989.

____. *Professional Standards for Teaching Mathematics*. Reston, Va.: The Council, 1991.

Madison, John P., and Roslynn Seidenstein. *Beyond Numbers: The Mathematics Literature Connection*. Reston, Va.: Mathematics Education Trust, 1987. (Available from the National Council of Teachers of Mathematics.)

Radebaugh, Muriel Rogie. "Using Children's Literature to Teach Mathematics." *Reading Teacher* 34 (May 1981): 902–6.

Smith, Nancy J., and Karla Hawkins Wendelin. "Using Children's Books to Teach Mathematics Concepts." *Arithmetic Teacher* 29 (November 1981): 10–15.

Thiessen, Diane, and Margaret Matthias. *The Wonderful World of Mathematics: A Critically Annotated List of Children's Books in Mathematics*. Reston, Va.: National Council of Teachers of Mathematics,1992.

Tischler, Rosamond Welchman. "Mathematics from Children's Literature." *Arithmetic Teacher* 35 (February 1988), 42–47.

Treffers, Adrian. *Three Dimensions: A Model of Goal and Theory Description in Mathematics Instruction—the Wiskobas Project*. Boston: D. Reidel Publishing Co., 1987.

Tunnell, Michael O., and James S. Jacobs. "Using 'Real' Books: Research Findings on Literature-based Reading Instruction." *Reading Teacher* 4 (March 1989): 470–77.

Wheeler, Margariete Montague. *Mathematics Library—Elementary and Junior High School*. Reston, Va.: National Council of Teachers of Mathematics, 1986.

Young, Sharon. *Mathematics in Children's Books: An Annotated Bibliography—Preschool through Grade 3*. Palo Alto, Calif.: Creative Publications, 1979.

BOOKS FOR CHILDREN

Note: This list includes all the children's books for which specific classroom activities are suggested in this booklet. The page numbers refer to pages in this booklet. The listing should not be regarded as a bibliography of books that *might* be suitable for classroom use in teaching mathematics but rather as a starting point for a way of thinking about making a mathematics-literature connection. See also the reference list in chapter 8 on page 72.

Anno, Masaichiro, and Mitsumasa Anno. *Anno's Mysterious Multiplying Jar*. New York: Philomel Books, 1983 60

Anno, Mitsumasa. *All in a Day*. New York: Philomel Books, 1986. 60

_____. *Anno's Counting Book*. New York: Crowell, 1977 . 37

_____. *The King's Flower*. New York: G. P. Putnam's Sons, 1970 54

Bang, Molly. *Ten, Nine, Eight*. New York: Greenwillow Books, 1983. 55

_____. *The Paper Crane*. New York: Mulberry Books, 1985. 10

Bayley, Nicola. *The Patchwork Cat*. New York: 1981 . . . 11

Berenstain, Stan, and Janice Berenstain. *Bears on Wheels*. New York: Random House, 1969 55

Brown, Marcia. *Stone Soup, an Old Tale*. New York: Aladdin Books, 1986 6

Burns, Marilyn. *The I Hate Mathematics! Book*. Boston: Little, Brown & Co., 1975 48

_____. *Math for Smarty Pants*. Boston: Little, Brown & Co., 1982 48

Carle, Eric. *My Very First Book of Shapes*. New York: Crowell, 1974 39

_____. *The Very Hungry Caterpillar*. New York: Putnam Publishing Group, 1989 54

Carroll, Lewis. *Alice's Adventures in Wonderland*. New York: Scholastic, 1985 48

_____. *Through the Looking Glass and What Alice Found There*. New York: St. Martin's Press, 1972 3

Coerr, Eleanor. *Sadako and the Thousand Paper Cranes*. New York: Putnam Publishing Group, 1977 10

Crews, Donald. *Freight Train*. New York: Greenwillow Books, 1978. 24

Dahl, Roald. *Charlie and the Great Glass Elevator*. New York: Alfred A. Knopf, 1972. 53

_____. *Matilda*. New York: Viking Penguin, 1988. 44

_____. *The Witches*. New York: Farrar Straus Giroux, 1983. 44

dePaola, Tomie. *Pancakes for Breakfast*. New York: Harcourt, Brace, Jovanovich, 1978 9

de Regniers, Beatrice Schenk. *So Many Cats!* Illustrated by Ellen Weiss. New York: Clarion Books, 1985 . . . 57

Douglas, Barbara. *The Chocolate Chip Cookie Contest*. New York: Lothrop, Lee & Shepard Books, 1985 10

Dr. Seuss. *The 500 Hats of Bartholomew Cubbins*. New York: Scholastic, 1966 65

_____. *One Fish, Two Fish, Red Fish, Blue Fish*. New York: Random House, 1987 23

Emberley, Ed. *Ed Emberley's Picture Pie, a Circle Drawing Book*. Boston: Little, Brown & Co., 1984 . . . . . 38, 60

Flournoy, Valerie. *The Patchwork Quilt*. New York: Dial Books for Young Readers, 1985 11

Frédérique and Papy. *Graph Games*. Illustrated by Susan Holding. New York: Thomas Y. Crowell Publishers, 1971 3

Fritz, Jean. *And Then What Happened, Paul Revere?* New York: Putnam Publishing Group, 1973. 48

Gag,Wanda. *Millions of Cats*. New York: Putnam Publishing Group, 1977 21

Galdone, Paul. *The Little Red Hen*. New York: Scholastic, 1973. 54

Gelman, Rita Golden. *More Spaghetti, I Say!*.Illustrated by Jack Kent. New York: Scholastic, 1977 16

Gentry, Tony. *Jesse Owens*. New York: Chelsea House, 1990 48

The Gingerbread Man. Any version 54

Goldilocks and the Three Bears. Any version 54

Hoban, Lillian. *Arthur's Funny Money*. New York: Harper & Row, 1981 . . . . . 10, 63

Hutchins, Pat. *Clocks and More Clocks*. New York: Young Readers Press, 1973 60

_____. *The Doorbell Rang*. New York: Greenwillow Books, 1987. . . . . 50, 60

Jackson, Holbrook, ed. *The Complete Nonsense of Edward Lear*. New York: Dover Publications, 1951 65

Jeffers, Susan. *All the Pretty Horses*. New York: Scholastic, 1985 23

Johnston, Tony. *The Quilt Story*. New York: Putnam Publishing Group, 1985 11

Jonas, Ann. *The Quilt*. New York: Greenwillow Books, 1984 11

____. *Reflections*. New York: Greenwillow Books, 1987. 51

____. *Round Trip*. New York: Scholastic, 1983. 51

Keats, Ezra Jack. *Over in the Meadow*. New York: Scholastic, 1971. 11

Kenneway, Eric. *Complete Origami*. New York: St. Martin's Press, 1987 10

Kent, Jack. *The Twelve Days of Christmas*. New York: Scholastic, 1971 . . . . . 11, 43

LeSieg, Theo. *Ten Apples Up on Top*. New York: Random House, 1961 13

Lindgren, Astrid. *Pippi Longstocking*. New York: Viking Penguin, 1978. 3

Lionni, Leo. *Swimmy*. New York: Alfred A. Knopf, 1987 23

Lobel, Arnold. *Frog and Toad Are Friends*. New York: Harper & Row, 1970. 5

McGovern, Ann. *Too Much Noise*. Boston: Houghton Mifflin, 1967 54

Mack, Stanley. *Ten Bears in My Bed*. New York: Pantheon, 1974. . . . . 11, 23

Mathis, Sharon Bell. *The Hundred Penny Box*. Illustrated by Leo and Diane Dillon. New York: Puffin, 1986. . . 9

Mori, Tuyosi. *Socrates and the Three Little Pigs*. Illustrated by Mitsumasa Anno. New York: Philomel Books, 1986 48

Myllar, Rolf. *How Big Is a Foot?* New York: Atheneum, 1962 58

Newell, Peter. *Topsys and Turvys*. New York: Dover Publications, 1964. 52

Norton, Mary. *The Borrowers*. New York: Harcourt Brace Jovanovich, 1953. 48

Nozaki, Akihiro. *Anno's Hat Tricks*. Illustrated by Mitsumasa Anno. New York: Philomel Books, 1985. . 48

Pittman, Helena Clare. *A Grain of Rice*. New York: Hastings House, 1986. 46

Potter, Beatrice. *The Tale of Squirrel Nutkin*. London: Frederick Warne, 1987 25

Potter, Marian. *The Little Red Caboose*. Illustrated by Tibor Gergeley. Racine, Wis.: Golden Books, 1978 24

Schwartz, David M. *How Much Is a Million?* Pictures by Steven Kellogg. New York: Lothrop, Lee & Shepard Books, 1985. 64

____. *If You Made a Million*. New York: Scholastic, 1989. 60

Silverstein, Shel. *Where the Sidewalk Ends*. New York: Harper & Row Publishers, 1974 65

Slobodkina, Esphyr. *Caps for Sale*. New York: Scholastic, 1984 16

Spohn, Kate. *Clementine's Winter Wardrobe*. New York: Orchard Books Watts, 1989 22

____. *Ruth's Bake Shop*. New York: Orchard Books Watts, 1990 22

Swift, Jonathan. *Gulliver's Travels*. New York: Putnam Publishing Group, 1947 48

Ungerer, Tomi. *One, Two, Where's My Shoe?* New York: Harper & Row, 1964. 39

Viorst, Judith. *Alexander Who Used to Be Rich Last Sunday*. Illustrated by Ray Cruz. New York: Atheneum, 1978. 65

Walter, Marion. *Another, Another, Another and More*. London: Andre Deutsch, 1975 52

____. *Magic Mirror Book*. New York: Scholastic, 1970 . . 52

____. *Magic Mirror Tricks*, New York: Scholastic, 1971 . 52

Wescott, Nadine Bernard. *I Know an Old Lady Who Swallowed a Fly*. Boston: Little, Brown & Co., 1981. . 11

White, E. B. *Stuart Little*. New York: Harper, 1945 48

White, Laurence B., and Ray Broekel. *Math-a-magic, Number Tricks for Magicians*. Illustrated by Meyer Seltzer. Niles, Ill.: Albert Whitman & Co., 1990 48

Williams, Vera B. *A Chair for My Mother*. New York: Greenwillow Books, 1982. 10

Wood, Gerald L. *Guinness Book of Pet Records*. London: Guinness Books, 1984. 46

Zaslavsky, Claudia. *Count on Your Fingers African Style*. Illustrated by Jerry Pinkney. New York: Thomas Y. Crowell Publishers, 1980 24

Zion, Gene. *The Meanest Squirrel I Ever Met*. New York: Atheneum, 1982 24

This pamphlet is part of the

NCTM "How to . . ." Series

These booklets are of a practical nature and are written in nontechnical language. Each title is intended to be directly applicable to the teaching of mathematics and have immediate classroom use.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

BEST COPY AVAILABLE